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THE INFLUENCE OF THE PROXIMITY OF A SOLID WALL ON THE CONSISTENCY OF VISCOUS AND PLASTIC MATERIALS*

BY R. K. SCHOFIELD AND G. W. SCOTT BLAIR

Introduction

In attempting to derive an expression for the rate of flow of a viscous or plastic material through a straight narrow tube of uniform cross-section under a pressure gradient, it is usually assumed:

(1) that each particle of the material moves with constant velocity in a straight line parallel to the axis.

(2) that there is no slip at the wall of the tube.

(3) that the velocity gradient at any point depends only on the shearing stress at that point.

Using these assumptions, it is shown below that, no matter how complex the relationship between velocity gradient and shearing stress (so long as the former is fixed when the latter is fixed), the volume extruded in unit time will depend, for a given stress at the wall of the tube, upon the cube of the radius. While this is true for fluids, and is also true or nearly true for thick pastes of soil and other minerals, it is found not to be generally true of such pastes when examined over an extended range of concentration. Discrepancies would occur if condition (1) were invalidated owing to turbulence; but reasons are given for considering it unlikely that turbulence is responsible for the effect. It is found that the mean velocity, instead of being proportional to the radius, is divisible into two parts, one proportional to it and the other independent of it. The second term apparently represents a velocity imparted to the bulk of the material by an excessive velocity gradient near the wall of the tube, suggesting that the proximity of a solid wall influences the consistency of these materials and causes a breakdown of condition (3). On subtracting the second term, the contribution made to the mean velocity by the flowing of the bulk of the material is presumably left, from which consistency constants relating to the material in bulk can be obtained independent of the dimensions of the tube.

Theoretical.

If condition (1) of the introduction be granted, so that the particles are not accelerated, the stress, W, at the wall of a tube of length L and radius R to which a pressure difference P is applied is PR/2L; while the stress, S, at a point T within the tube and distant r from the axis is Pr/2L. Consequently r can be expressed in terms of S thus:

$$r = R/W.\ S. \qquad (1)$$

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If v be the velocity at T, we may write in accordance with condition (3)

$$dv/dr = -f(S).$$

Substituting the value of r given by equation (i), and integrating

$$v = R/W \int_s^W f(S)dS, \tag{ii}$$

if, in accordance with (2) v = 0 when S = W. The flow dV, between r and r + dr = 2πdr.v. Substituting for r and v from equations (i) and (ii) and integrating,

$$\frac{V}{\pi R^3} = \frac{2}{W^3}\int_0^W S \int_s^W f(S)dS.dS \tag{iii}$$

From this it is clear that, for any given material, $V/\pi R^3$ should depend only on W if the three conditions are fulfilled.

By making specific assumptions about the form of f (S), $V/\pi R^3$ can be evaluated. Thus using the Maxwell assumption that

$$f(S) = \mu S$$

where μ (the fluidity) is the reciprocal of the viscosity, equation (iii) reduces to Poiseuille's equation in the form

$$\frac{V}{\pi R^3} = \frac{1}{4}\mu W.$$

In the same way the expression based on the Bingham[1] assumption can be deduced. Here it is supposed that the material does not flow unless a stress exceeding a critical value, S_o, be applied to it and that, at stresses higher than S_o, the velocity gradient equals μ (S − S_o). Again μ is a constant having the dimensions of a reciprocal viscosity, and is usually called the mobility. When such a material is forced through a tube, a central cylinder of radius RS_o/W, within which the stress is less than S_o, moves as a solid plug, and only the material outside this cylinder flows. When W is less than S_o, no flow occurs, and V = 0. In substituting in equation (iii) to obtain the value of V when W exceeds S_o, it must be remembered that, since f(S) is discontinuous, being zero from 0 to S_o and μ (S − S_o) from S_o to W, the integrations must be carried out in two stages. This has the effect of splitting V into two terms, thus

$$\frac{V}{\pi R^3} = \frac{2}{W^3}\int_0^{S_o} S \int_{S_o}^W \mu(S-S_o)dS.dS + \frac{2}{W^3}\int_{S_o}^W S \int^W \mu\,(S-S_o)dS.dS.$$

The first is the contribution of the plug, the second is that of the flowing material between it and the wall. This reduces to

$$\frac{V}{\pi R^3} = \frac{1}{4}\mu\left[W - \frac{4}{3}S_o\left\{1 - \frac{1}{4}\left(\frac{S_o}{W}\right)^3\right\}\right]$$

which is subject to the condition that W > S_o. This is equivalent to the

[1] E. C. Bingham: "Fluidity and Plasticity," (1922).

equation[1] deduced by Buckingham,[2] and independently by Reiner.[3] The graph connecting $V/\pi R^3$ and W corresponding to this equation is tangential to the W axis at $W = S_o$. It is strongly curved for values of W only slightly exceeding S_o, but at higher values approximates to a straight line of equation

$$\frac{V}{\pi R^3} = \frac{1}{4}\,\mu(W - \frac{4}{3}\,S_o).$$

This line makes an intercept on the W axis equal to $4/3 . S_o$, and, like the corresponding graph of the Poiseuille equation, has a slope of $1/4. \mu$. The true curve is steeply asymptotic to the limiting straight line, the discrepancy in V being less than 1% when W exceeds 2.2 times the intercept of the limiting straight line.

If the parabolic relation

$$f(S) = \bar{\mu}\,S^n$$

of the Ostwald type[4] be assumed, equation (iii) reduces to

$$\frac{V}{\pi R^3} = \frac{1}{n+3}\,.\,\bar{\mu}\,.W^n$$

which is equivalent to the equations given by Farrow, Lowe and Neale,[5] and Porter and Rao.[6] The constant $\bar{\mu}$ has dimensions which depend on the exponent n. The graph in this case starts from the origin, and is curved throughout its length. The curvature is never very strong, but it decreases only slowly with increasing W.

Experiments.

The modified Bingham plastometer used in this work has already been described.[7] The paste to be investigated is made by mixing the soil, clay or other mineral with water into a smooth paste, which is then forced through a one hundred mesh-per-inch sieve to remove any coarse particles. The paste is then diluted to the required concentration, and sucked into the plastometer bulbs which, for the high-stress work here described, have a capacity of 100 c.c. each. The material is forced alternately from one bulb into the other through one of a series of standardised tubes, the level in the bulbs being kept approximately the same by tilting the whole system about a pivot. For this more accurate work with larger bulbs it becomes necessary to correct the pressures for the resistance offered by the bulbs themselves. For this purpose the bulbs are connected directly with one another, and the pressures corresponding to a series of volume-flows are measured. By graphi-

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[1] Buckingham's equation contains an additional term which is negligibly small when W exceeds S_o and which is referred to below.

[2] E. Buckingham: J. Am. Chem. Soc., Test. Mat., **1921**, 1154.

[3] Reiner: Kolloid-Z., **39**, 80 (1926), etc.

[4] Wo. Ostwald (and others): Kolloid-Z., **36**, 99, 157, 248 (Zsigmondy Festschrift) 252 (1928); **38**, 261 (1926); **41**, 56, 112 (1927). (This type of equation was, of course, not originated by Ostwald, but he has made much use of it.)

[5] Farrow, Lowe and Neale: J. Textile Inst., **19**, T 18 (1926).

[6] Porter and Rao: Trans. Faraday Soc., **23**, 311 (1927).

[7] G. W. Scott Blair and E. M. Crowther: J. Phys. Chem., **33**, 321 (1929).

cal intrapolation the correction, Pb, corresponding to each volume-flow can be estimated. As no appreciable increase in resistance is caused by introducing a few millimeters of narrow tubing between the bulbs, it may safely be concluded that no kinetic energy correction is necessary with these measurements. The tubes had been carefully selected with a view to uniformity of bore and were standardised by weighing the quantity of mercury required to fill them. A series of constant pressures are applied by means of compressed air, the pressure being measured on a water or mercury manometer according to its magnitude. The air displaced by the clay is allowed to escape through an air-capillary of suitable dimensions. The pressure difference (negligible in comparison with the applied pressure) is measured on an alcohol manometer at an angle of one in ten (the flow-meter), and is directly proportional to the volume of flow of paste per second. The moisture content of the paste is determined by heating a sample for one hour in an oven at a temperature of 160°C. The concentration, K, is expressed as the number of grams of dry matter per 100g paste. Volume concentrations are calculated on the basis of a constant specific gravity for the dry material of 2.7.

In carrying out this work efforts have been made to use as wide a range of radii as possible. Although it is hoped in the future to increase this still further, difficulties will first have to be overcome. Thus the use of very wide capillaries involves large volumes of material and consequently big bulb correction (always difficult to determine accurately). Moreover the increased length that must be given to the tube necessarily entails a sacrifice of uniformity in the bore. Thus beyond certain limits, the loss in accuracy renders further increase in radius of no advantage. With very narrow tubes so coarse a system as a soil-paste behaves erratically.

In Table I are given, as an example, the complete data for a sample of Broadbalk Field subsoil similar to that of which the clay-fraction has been used in the previous work.

TABLE I

Plastometric data for Broadbalk subsoil paste (33.5 g dry soil per 100 g paste).

Cap. II. R = 0.093 cm. L = 12.20 cm.

P	a	V	*P*b	S	v.
8.0	3.0	2.0	1.1	3.5	.75
9.0	4.2	2.7	1.3	3.9	1.05
10.0	4.4	3.0	1.4	4.4	1.10
11.0	5.2	3.5	1.5	4.9	1.30
12.0	5.7	3.7	1.5	5.4	1.42
13.0	6.7	4.6	1.8	5.7	1.68
14.0	7.0	4.8	1.8	6.2	1.75
15.0	8.2	5.4	1.9	6.7	2.05

Cap. III. R = 0.073 cm. L = 12.10 cm.

P	a	V	Pb	S	v.
14.0	2.8	1.9	1.0	5.2	1.13
16.0	3.7	2.5	1.3	5.9	1.49
20.0	4.2	2.9	1.4	7.4	1.73
22.0	5.1	3.5	1.4	8.2	2.08
24.0	5.7	3.9	1.5	9.0	2.32
26.0	6.1	4.2	1.6	9.7	2.50
28.0	6.8	4.6	1.8	10.4	2.74
30.0	7.3	5.0	1.8	11.2	2.97

Cap. 7 R = 0.059 cm L = 10.60. cm

P	a	V	Pb	S	v.
15.0	1.5	1.0	0.7	5.3	0.93
20.0	2.4	1.6	1.0	7.1	1.48
25.0	3.2	2.0	1.1	8.9	1.98
30.0	4.0	2.6	1.3	11.1	2.50
35.0	4.8	3.1	1.4	12.5	3.00

Cap. IV. R = 0.048 cm. L = 12.25 cm.

P	a	V	Pb	S	v.
16.0	0.5	.34	0.6	3.9	.47
24.0	1.05	.72	0.7	6.0	.99
28.0	1.3	.88	0.7	7.0	1.22
32.0	1.6	1.09	0.8	8.0	1.50
38.0	1.95	1.33	0.8	9.5	1.85
40.0	2.0	1.36	0.8	10.0	1.88
42.0	2.1	1.43	0.8	10.5	1.98
44.0	2.25	1.53	1.0	11.0	2.12

Cap. V. R = 0.040 L = 12.30 cm.

P	a*	V	Pb	S	v.
20.0	0.3	.06	0.6	4.2	.40
25.0	0.5	.10	0.6	5.3	.66
30.0	0.7	.13	0.6	6.4	.93
35.0	0.85	.16	0.6	7.4	1.13
40.0	1.05	.20	0.6	8.5	1.35
45.0	1.2	.23	0.6	9.6	1.60

Bulbs alone without capillary.
(A water manometer was used, but P is given converted into cm. Mercury).

P	0.5		1.0	1.5	2.0
a	*0.2	0	2.0	6.0	8.0
V	0.4	0	1.4	4.1	5.4

* (V/a = .19).

P is pressure in cm. mercury.
a is flowmeter reading (V/a = 0.68).
V is volume flow in cm^3/secs.
Pb is pressure due to bulbs in cm. mercury.
S is stress in dynes/$mm.^2$ (calculated from P − Pb).
v is mean velocity in metre/secs.

It is well known that the data obtained with such an apparatus when plotted on a $V/\pi R^3-W$ basis fall into two groups. In one, which may conveniently be called type A the points for a single tube lie, within the limits of experimental error, on a straight line passing through the origin. In the other

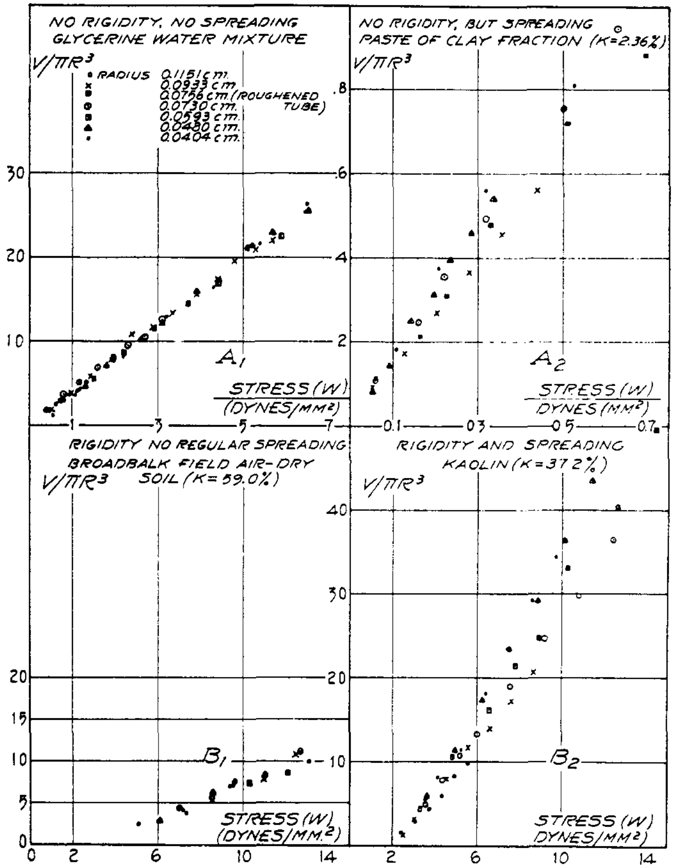

FIG. 1
Legends incorporated in four corners of each quarter of drawing.

(type B) this is not the case. Our own measurements not only confirm this fact but show that each group must be further subdivided according as the curve obtained is or is not independent of R. There are thus four possibilities, an instance of each which is given in Fig. 1.

With the water-glycerine mixture (type A1) a single straight line through the origin gives an adequate representation throughout the range of stress used. This is not true of the dilute suspension of very fine soil particles.

(type A2) Here the best line though passing through the origin has a larger slope the smaller the radius. Thus the disconcerting fact is here revealed that a straight line through the origin obtained with a single tube is not by itself a proof that Poiseuille's law is being obeyed.

The thick paste of Broadbalk Field surface soil gives points which, though more erratic than those for water-glycerine, show no regular trend with change of radius. Nevertheless they cannot be represented as falling on a straight line through the origin (type B1). The behaviour of *thick* soil pastes has already been described in detail in the earlier paper,[1] where it is shown that the curves can be interpreted in the light of the Bingham postulate. New and more accurate measurements over a wider concentration range has shown that a slight spreading noticeable in some of the earlier data and attributed to experimental error cannot be so explained. In thinner pastes of soils, clays and simple minerals such as barytes and gypsum the spreading is very marked. A kaolin paste of moderate consistency is given as an example (type B2). Here as with type A2 $V/\pi R^3$ for a given value of W increases as R decreases.

An alternative way is to regard type B2 as the general case: the other three being special and simpler cases. Such a view raises the question as to whether all these systems are susceptible to the same treatment, and can therefore be represented by a single though complex, equation. Already in the interpretation of curves of the B1 type, two distinct schools of thought have developed, one of which bases its treatment on Bingham's postulate and the other on the Ostwald postulate. This is not the place to enter into a general discussion of the relative advantages of the two methods, suffice it to say that all the soil and mineral pastes investigated in this laboratory are more amenable to the first method; and that although there undoubtedly are systems such as benzene-rubber and pastes of at least some starches that give curves of a shape not accounted for by the simple Bingham postulate, it is nevertheless true that much of the data which is represented as conforming to a relationship of the Ostwald type can as well be cited in support of the simple linear relationship. (Vide Herschel and Bulkley,[2] Porst and Moskowitz,[3] Scott Blair,[4] Ostwald[5]).

Hatschek[6] has criticised the practice of extrapolating flow-curves by means of straight lines, and considers that, failing a discontinuous change from a curved to a straight portion, the choice of the portion to be regarded as straight is arbitrary and a matter of scale. According to the Bingham treatment these straight lines are asymptotes to which the true flow-curve approximates more and more closely as the stress increases. It is clear in general that the error involved in drawing an asymptote to an experimental curve depends on the

[1] G. W. Scott Blair and E. M. Crowther: J. Phys. Chem., **33**, 321 (1929).
[2] Herschel and Bulkley: Ind. Eng. Chem., **16**, 927 (1924) etc.; Proc. Am. Soc. Test. Mat., **26**, 621 (1926).
[3] Porst and Moskowitz: J. Ind. Eng. Chem., **14**, 49 (1922).
[4] Scott Blair: Kolloid-Z., **47**, 76 (1929).
[5] Ostwald: Kolloid-Z., **47**, 176 (1929).
[6] Hatschek: "The Viscosity of Liquids," 209 (1928).

steepness of approach. It would certainly be difficult to draw the asymptote to a rectangular hyperbola given only a portion of the curve; but the criticism loses its force where the curve is of the Buckingham-Reiner type. In this case, already noted, the discrepancy in V is less than 1% for values of W greater than 2.2 times the intercept. Above this limit the difference between the true curve and the limiting straight line should be outside the limits of experimental error. This is borne out in practice with soil and clay pastes, so that linear extrapolation appears justified with these materials.

Discussion.

These experiments yield the result that for many viscous suspensions (type A) as well as plastic pastes (type B), $V/\pi R^3$ does not depend only on W. As the variation of the former quantity sometimes approaches twofold for a twofold variation of radius, the effect is evidently quite outside the limits of ordinary experimental error. Moreover the fact that the apparatus gives a very satisfactory verification of Poiseuille's law for true fluids indicates that it works satisfactorily. The effect has every appearance of being genuine. The next step therefore is to seek its cause. This must lie in a breakdown of one or more of the three conditions set out in the introduction. These will be considered in turn.

A breakdown of the condition that the particles move with a constant velocity parallel to the axis would occur if the flow were turbulent. Although no complete theory of turbulent flow has yet been advanced, it is generally considered that its presence is marked by a falling off in the slope of the flow-curve as the stress is increased. Such a falling off does occur at very high stresses, particularly with the more dilute suspensions; but the curves obtained with a view to elucidating the effect under discussion were not followed far enough for this to happen, and as can be seen from the examples given, show no signs of curvature over the range examined. It might be urged that the close approximation to linearity arises from a chance cancellation of opposite curvations, in a manner similar to that suggested to explain the "Laminarast" or linear portion of the flow-curves obtained by Ostwald and Auerbach.[1] As against this, it should be pointed out that many hundreds of flow-curves for clay and soil pastes have now been accumulated in this laboratory, and not one reliable curve has been obtained which cannot be fairly represented by a straight line at sufficiently high stresses. It seems inconceivable that an exact cancellation of two unconnected tendencies should occur in all these cases. It is more reasonable to interpret the straightness as an indication that these materials are obeying both condition (1) and the Bingham postulate and to endeavour to deduce an equation of the Buckingham-Reiner type based on a modification of conditions (2) and (3).

Until the results to be expected when flow is turbulent have been more fully worked out, it is impossible to exclude it altogether from the possible causes contributing to the effect. All that can be said at present is that there

[1] Ostwald and Auerbach: Kolloid-Z., **38**, 261 (1926); **41**, 56 (1927).

is no positive evidence that turbulence is present in these experiments. This statement applies with equal force to the 'structure' type of turbulence postulated by Ostwald as to the more general type.

On plotting the mean velocity $V/\pi R^2$ rather than $V/\pi R^3$ against W, a regularity becomes apparent. A typical set of curves plotted in this way

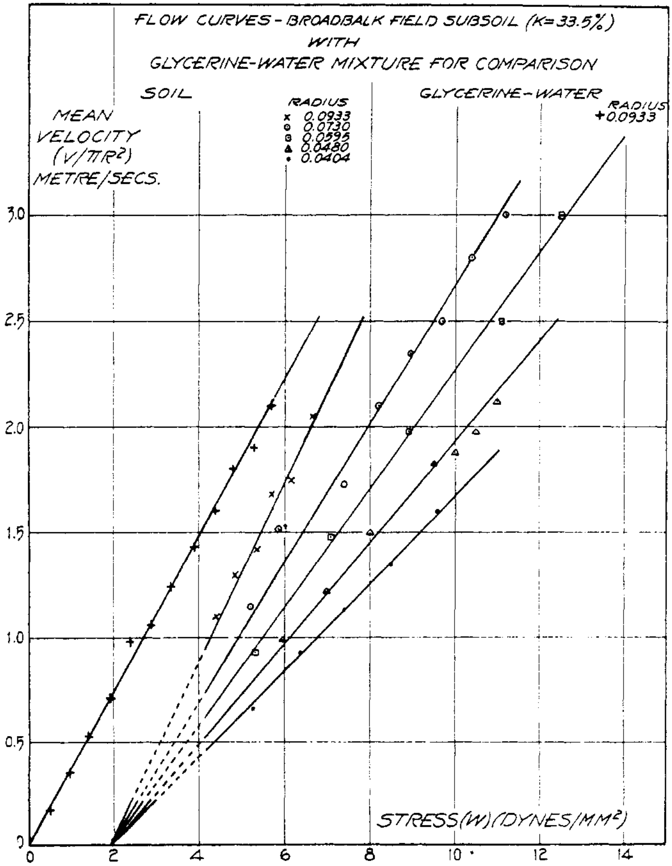

FIG. 2

from the data in the table is shown in Fig. 2. Since according to the Buckingham-Reiner equation (see above) the points should lie above the limiting straight line by more than 1% at values of W less than 2.2 times the intercept these have been omitted, and only those for stresses above 4.4 dynes/mm² have been used for locating the limiting straight lines. As these have a common intercept their slopes would be proportional to R were $V/\pi R^3$ dependent only on W. Actually when the slopes σ are plotted against R a straight line can in all cases be drawn through the points, but for types A2 and B2 this does not, when extrapolated, pass through the origin, but gives a positive intercept on the slope axis. Curves connecting σ and R which may conveniently be spoken of as derived curves are given in Fig. 3 for the four sets of curves of Fig. 1. The derived curve obtained from Fig. 2 together with ones for pastes of barytes and gypsum are given in Fig. 4. If turbulence

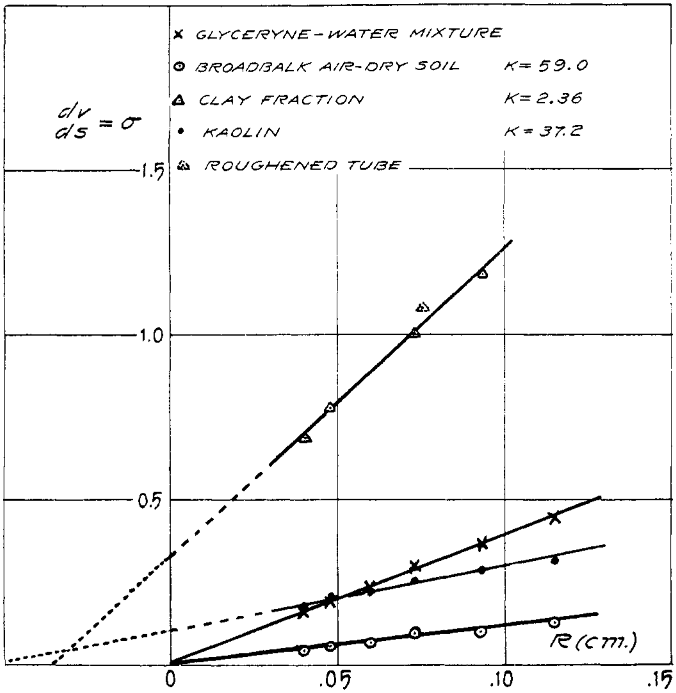

FIG. 3
Derived curves

were the sole cause of the effect, the disturbance would presumably be most marked in the widest tube. In these conditions the mean velocity for a given stress on the wall should be more nearly proportional to the radius with the smaller tubes than with the wider ones. In other words the derived curve should approximate more closely to a straight line through the origin the smaller the radius. It will be seen that the curves in Figs. 3 and 4 taken as a whole do not support this idea.

If, on the other hand, the derived curves are in reality straight lines (as shown), the fact that some give an intercept might be interpreted as indicating that conditions (1) and (3) are fulfilled, but that in such cases there is a slip at the wall. The slope σ, and hence the mean velocity at a given value of W can in these cases be separated into two components, one proportional to the radius and one independent of it. It seems more probable however that the second component instead of representing an actual slip at the wall should be regarded as a velocity imparted to the bulk of the material by excessive flowing of the material in the immediate vicinity of the wall. Provided that the thickness of the region in which this excessive flow takes place is both independent of the radius and also small in comparison with it, the effect on

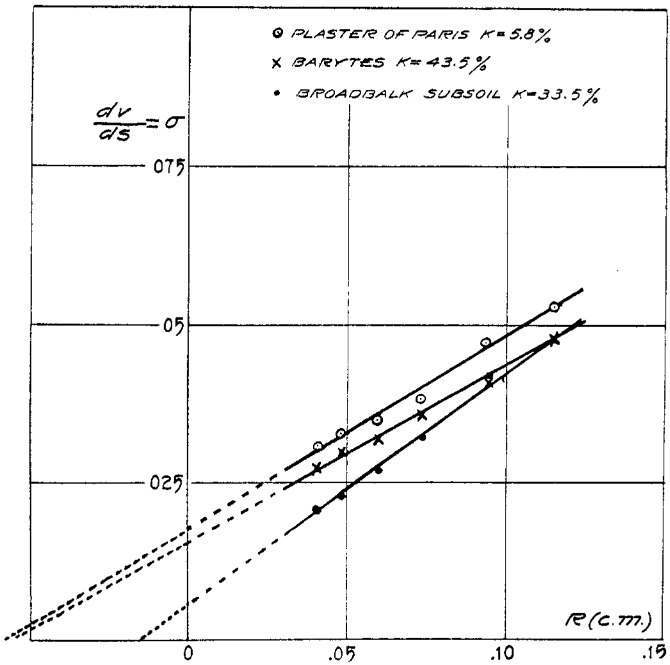

FIG. 4
Derived Curves

the mean velocity would be the same as that of a slip at the wall itself. There would however be a difference in tubes so narrow that the radius is of the same order of magnitude as the thickness of the region of excessive flow, as in this case the derived curve would bend round towards the origin.

The first component of the mean velocity, since it is proportional to R is presumably due to the flow of the material in bulk, and therefore equal to $V/\pi R^2$ calculated from equation (iii) using the appropriate value for f(S). In cases where the Bingham postulate is applicable to the material in bulk the first component will be that given by the Buckingham-Reiner equation and the slope of the derived curve will equal $\frac{1}{4}\mu$. The second component of mean velocity is equal, according to the constructions in Figs. 2 and 3, to σ_0 (W−C) when σ_0 is the intercept of the derived curve on the σ axis and C the common intercept of the flow curves (Fig. 2) on the W axis (which according to the Buckingham-Reiner equation should equal $4/3\ S_0$)

The existence of a very thin layer of fluid of the consistency of water, separating the paste from the wall has been assumed by Buckingham to account for the small movement which occurs at stresses so low that the bulk of the material does not flow. This view, slightly modified, was adopted in the earlier paper, where it was shown experimentally that this flow is related to the stress thus:

$$V/\pi R^2 = v = \epsilon\phi\ (W-A) \qquad \text{(vii)}$$

A being a constant stress below which no movement occurs, ϵ the thickness of the layer and ϕ its mobility. It might at first appear that the second component of the velocity is the same as the above. This, however, can hardly be the case since σ_0 is found to have a magnitude some 100 times that of $\epsilon\phi$. For this reason the second component, unlike the Buckingham term, cannot be neglected at high stresses when the material is flowing. Assuming a value of ϕ equal to the fluidity of water in bulk, the thickness ϵ is of the order 10^{-5}cm. The corresponding thickness calculated from σ_0 is of the order 10^{-3}cm As the mobility of the modified layer cannot be greater than that of water in bulk and is probably less, this latter is a minimum estimate.

Experiments with tubes, the walls of which had been etched with fluoride also reveal an essential difference between the two phenomena. It was found (loc. cit.) that etching greatly interferes with the motion at very low stresses and renders equation (vii) inapplicable. No corresponding influence on σ_0 is observed. Thus it will be seen from Figs. 1 and 3, that the points obtained with the etched tube fall into line with those for the smooth tubes. A further distinction is apparent in the behaviour of pastes made from soils that have previously been air-dried. Movement at low stress is inhibited whereas no similar interference is found at high stresses. It would appear therefore that, where the derived curve does not pass through the origin, there exsits in the immediate neighbourhood of the wall of the tube a region in which the viscous or plastic properties of a material flowing through it are modified. When a laminated material such as a clay paste flows through a tube particles near the wall will tend to align themselves in the direction of flow, and be unable to rotate under the influence of the viscous couple acting in them. In the bulk of the material the particles will rotate sufficiently to prevent any alignment. If indeed there be any such an orientation near the wall it might give rise to an increase in mobility as the wall is approached and thus to a deviation from equation (iii) such as is actually observed. On the other hand attempts to eliminate the σ_0 term by the use of materials which are believed to be devoid of laminar structure has so far proved unsuccessful. Only true fluids show both no σ_0 term and no rigidity. An alternative explanation would involve an increased concentration of the suspended material towards the centre of the tube relative to the region near the wall, the relatively more dilute material having a greater mobility. In order to test this idea a sample of clay suspension giving a high value for σ_0 was forced through a metal tube through the side of which a hole had been drilled. This hole was very small so that the exuding of material through it would scarcely interfere with the flow in the tube. Although a large variation in concentration would have to be assumed to account for the value of σ_0 observed, no appreciable difference n concentration was found between the exuded material and the rest.

We are not, therefore, in a position to offer a detailed physical explanation of the effect observed. Yet the view that the properties of the material are modified in the neighbourhood of the wall is supported by another fact which has been repeatedly observed, but which has hitherto received no explanation. It will be seen that in Fig. 2 of the previous paper (p. 326) the ratio of the extrapolated intercept on the pressure axis in the V-P curve, to the pressure at which flow at the wall just starts, is somewhat greater than the 4/3 necessitated by the Buckingham-Reiner equation. This discrepancy may well be due to a decreased value of S_0 in the neighbourhood of the wall.

The Determination of Consistency Constants.

Methods for determining absolute consistency constants have hitherto been based on the supposition that, provided the motion is not turbulent, $V/\pi R^3$ depends only on W and the nature and concentration of the material. Where this is not true (Types A2 and B2) the methods require modification whether the constants are defined with reference to the curves obtained by plotting $V/\pi R^3$ against W (Buckingham-Reiner equation) or by plotting the logarithms of these quantities (Farrow, Lowe and Neale equation). In such cases, it is impossible to define a viscous constant for the material from measurements made with a single capillary, since there would be a progressive change in its value with radius were the usual method adopted. An instance is given in Fig. 5 where the viscosity as given (1) by the Poiseuille-Bingham method for wide and narrow capillaries respectively and (2) from the slope of the derived curve is plotted against the concentration. The latter construction has the advantage of giving a single constant independent of radius, which, if the considerations advanced above are correct is a measure of the viscous properties of the bulk of the material.

Moreover, the shape of the viscosity-concentration curve as given by the second construction is such as would be expected if the deviation from linearity were due simply to a decrease in the extent of hydration of the particles. This shape is not reproduced in the curves derived from the single capillaries. The curve from the smallest capillary has an upward curvature of the type frequently obtained for concentration curves for hydrophylic materials with a simple Ostwald viscometer.

It is at least apparent that measurement of the viscosity of suspensions cannot be considered to be reliable unless made with at least two tubes of reasonably differing radii.

The method at present used in this laboratory for determining the consistency constants for soil and clay pastes is as follows:

The data for a series of four or five different capillaries having a total range of radius of at least two-fold are plotted out on a $V/\pi R^2{:}PR/2L$ basis, the values of P having previously been corrected for the resistance of the bulbs. The best straight lines are then drawn through the points in such a way that they converge on the stress axis. The point at which these extrapolations converge is taken as the rigidity, C.[1] The slope (σ) of each curve is then measured (i.e. the rise in $V/\pi R^2$ per unit increase in stress) and σ

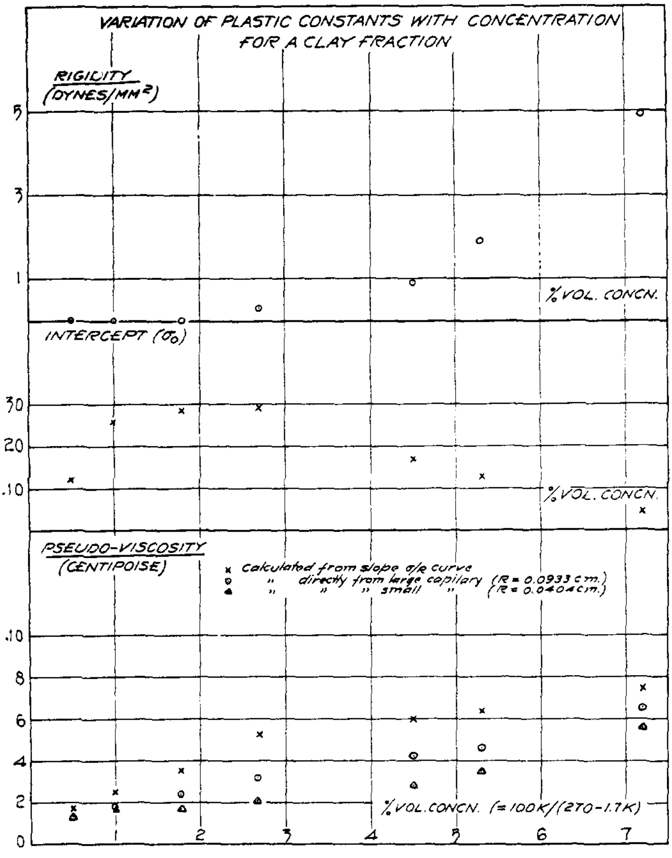

FIG. 5

is plotted separately against R. The slope of the derived curve ($dR/d\sigma$) divided by 4 is taken as the viscous constant η' (pseudo-viscosity $= 1/\mu$) and the intercept of the extrapolated curve on the σ axis as σ_0, a measure of the wall effect.

Fig. 5 shows an interesting relationship for a clay fraction between σ_0 and concentration, the value of σ_0 passing through a maximum at a low

[1] This is, of course, the "limit of rigidity" i.e. Bingham's yield value, not the rigidity modulus.

concentration and disappearing for pure water and also at high concentrations where the high rigidity limits a further extension of the concentration range. Accurate measurements of σ_0 at low concentrations are not easy, and the values are liable to a fairly large error, but there can be no doubt as to the general shape of the curve. It is of interest that the maximum of the curve occurs at about the same concentration as that at which rigidity first makes its appearance.

Since the dimensions of σ_0 are somewhat inconvenient, an alternative method is to extrapolate the derived curves still further onto the negative radius axis. In this way a hyperthetical length (R_0) is obtained which must be added to each radius before the equation stating the proportionality of the slope of the $V/\pi R^2$: $PR/2L$ curves to (viscosity $\times$ radius) can be applied. In other words $R^3(R + R_0)$ takes the place of the R^4 in the equations of Poiseuille or Buckingham-Reiner when written so as to give V directly in terms of P.

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Summary

If, in considering the flow of a plastic material through a narrow tube, it be assumed that the velocity gradient at any point depends only on the stress at that point, it necessarily follows that the mean velocity for a given stress at the wall of the tube should be directly proportional to the radius of the tube. Although thick soil pastes conform closely to this requirement, thinner pastes whether they show rigidity or not give marked discrepancies. These discrepancies can be accounted for by assuming that in the immediate proximity of the wall a modification of the plastic properties occurs, which imparts an additional velocity to the bulk of the material. By first subtracting this velocity a viscosity constant is obtained independent of the dimensions of the tube.