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THE ANALYSIS OF A FACTORIAL EXPERIMENT WITH ADDITIONAL TREATMENTS

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When planning a factorial experiment, it is often desirable (as pointed out by Yates (1937)) to include certain extra treatments falling outside the usual factorial scheme. I discuss here the exact analysis of an experiment of this type. Only standard principles of experimental analysis are involved, and the full exact analysis is not always required in practice, but the example may illustrate techniques that are useful in investigating more complicated designs that are sometimes encountered. I am grateful to the late Dr E. M. Crowther and Miss B. Benzian, of the Chemistry Department at Rothamsted, for discussion of the problem and for providing data for the illustrative example.

Table 1. *Plan and results of experiment*

—	NP	C	C	PK	NP
7·0	11·2	8·7	11·1	11·0	14·0
PK	(1)	NK	NK	—	(1)
8·4	6·7	12·8	14·4	10·1	9·4
P	—	K	NPK	—	N
9·1	7·9	7·9	14·0	9·3	11·3
N	C	NPK	P	K	C
11·4	9·5	13·5	10·6	8·2	10·7
Replicate 1			Replicate 2		

Mean height of transplants in inches.

The experiment under discussion was on sitka spruce transplants, and was basically a 2^3 design testing nitrogen (N), phosphate (P) and potash (K), each present and absent (in fact a fourth factor was also included in the experiment, but this has been omitted to save space). The eight treatments in each replicate were arranged in two blocks of four plots each, confounding the three-factor interaction. In addition, two extra plots were included in each block, one untreated and the other receiving compost (C). These two plots were randomized in with the others, and the plan and results (average heights of transplants in inches) on two replicates are given in Table 1.

The simplest treatment is to keep the factorial experiment and the additional plots separate and to construct two analyses of variance of the form

I. Factorial treatments	D.F.	II. Additional treatments	D.F.
Blocks	3	Blocks	3
Treatments	6	Treatments	1
Error	6	Error	3
Total	15	Total	7

Since the additional treatments were randomized in with the others, the two estimates of error are comparable and can be pooled to give an error mean square with 9 D.F. This mean square can be used for assessing the precision of any contrast between the treatment means. However, we have allowed 6 D.F. in all for blocks, and these can be broken down in a more exact analysis to give two more D.F. for error and an estimate of the confounded three-factor interaction.

Table 2 gives, above the heavy line, a set of ten

Table 2. *Individual degrees of freedom*

	Treatment I								Treatment II				Replicate			
	(1)	*n*	*p*	*np*	*k*	*nk*	*pk*	*npk*	–	*c*	—	*c*	1	2	Sum	Diff.
1 N	–	+	–	+	–	+	–	+	.	.	.	.	+16·8	+14·5	+31·2	+2·3
2 P	–	–	+	+	–	–	+	+	.	.	.	.	+3·4	+6·3	+9·7	–2·9
3 NP	+	–	–	+	+	–	–	+	.	.	.	.	–2·4	–1·7	–4·1	–0·7
4 K	–	–	–	–	+	+	+	+	.	.	.	.	+4·2	+2·3	+6·5	+1·9
5 NK	+	–	+	–	–	+	–	+	.	.	.	.	+3·2	+3·9	+7·1	–0·7
6 PK	+	+	–	–	–	–	+	+	.	.	.	.	–1·0	–1·5	–2·5	+0·5
7 Blocks (I) ≡NPK	–	+	+	–	+	–	–	+	.	.	.	.	+2·8	–4·7	–1·9	+7·5
8 Blocks (II)	.	.	.	.	.	.	.	.	–	–	+	+	+1·7	–1·2	+0·5	+2·9
9 C	.	.	.	.	.	.	.	.	–	+	–	+	+3·3	+2·4	+5·7	+0·9
10 Error (C × Blocks)	.	.	.	.	.	.	.	.	+	–	–	+	–0·1	+0·4	+0·3	–0·5
11 I *v.* II	–	–	–	–	–	–	–	–	+2	+2	+2	+2	–14·8	–10·5	–25·3	–4·3
7*a* Blocks	–	+	+	–	+	–	–	+	–	–	+	+	+4·5	–5·9	–1·4	+10·4
8*a* NPK	–	+	+	–	+	–	–	+	+2	+2	–2	–2	–0·6	–2·3	–2·9	+1·7

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contrasts in a single replicate which can be used to build up the above analyses.

For a full analysis, we must include an eleventh contrast giving the difference between the mean of the factorial treatments and the mean of the additional treatments. We also replace contrasts 7 and 8 by the ordinary block contrast, 7*a*, derived from the block totals over all the treatments, and a further contrast, 8*a*, which is completely determined by the fact that it must be orthogonal to all the other contrasts. On examination, this is seen to be an estimate of the three-factor interaction, *ABC*. We can now take the sums and differences of these contrasts over the two replicates and obtain from the sums:

	D.F.	S.S.
Blocks	1	0·081667
Treatments:		
Factorial	7	74·518958
Others	1	4·061250
Factorial *v.* others	1	13·335208
Error	1	0·011250

and from the differences

Blocks × replicates	1	4·506667
Others × replicates	10	1·736667

and including the 1 D.F. between totals of replicates we arive at the final analysis

	D.F.	S.S.	M.S.
Blocks	3	21·2550	
Treatments	9	91·9154	
Error	11	1·7479	0·1589
Total	23	114·9183	

The three-factor interaction is, of course, partially confounded. From the sum of the squares of the coefficients of contrast 8*a*, we find that the variance of the corresponding total is $48\sigma^2$ in place of $16\sigma^2$ in an unconfounded experiment. Two-thirds of the information on this comparison is thus lost by the confounding, the remaining one-third being recovered by the full analysis.

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