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STUDIES IN CROP VARIATION.

VIII. AN APPLICATION OF THE RESISTANCE FORMULA TO POTATO DATA.

By R. J. KALAMKAR, B.Sc., B.Agr.

Statistical Department, Rothamsted Experimental Station, Harpenden.)

I. Introduction.

A mathematical expression called the Resistance Formula which formulates a yield-factor relationship as suggested by Maskell has been critically tested by Bhai Balmukand (1). The formula postulates that the reciprocal of the yield is the sum of portions, each a function of one nutrient, such as $F(N)$, $F'(K)$, etc. Further, the function of a particular nutrient is inversely proportional to that nutrient in an available form. Based on this hypothesis, the equation for the yield y can be represented thus:

$$\frac{1}{y} = F(N) + F'(K) + F''(P) + \ldots, \text{ etc.},$$

where

$$F(N) = \frac{a_n}{n+N}, \quad F'(K) = \frac{a_k}{k+K},$$

a_n, a_k, etc. being the constants varying with the nutrient and the crop; n and k represent the available nitrogen and potash in the unmanured soil, and N and K the quantities added.

II. Material and method of investigation.

The object of this paper is to make a further test of the validity of the Resistance Formula, and use is made of the Rothamsted Potato Experiment on Long Hoos Field, Section I, in 1929. This experiment was designed to give information as to the effect on yield of applying nitrogenous, potassic and phosphatic fertilisers in various quantities. There was a basal dressing of dung at the rate of 14 tons per acre and further nitrogen was supplied as sulphate of ammonia at rates of 0, 0·3 and 0·6 cwt. of nitrogen and potash in the forms of sulphate, muriate and potash manure salts at rates equivalent to 0, 0·5 and 1 cwt. of K_2O per acre. Superphosphate was applied at the rate of 0·4 cwt. of P_2O_5 per acre. The experiment consisted of 81 plots arranged in nine blocks each having plots treated in all combinations of nitrogenous and potassic manures in none, single and double dressings. Further, every block was equalised with respect to the different kinds of potassic fertilisers, *i.e.* the three plots having single potash had it in the forms of sulphate, muriate and potash salts respectively, and likewise the doubly dressed plots, and each set of these blocks, taken by row or by column, had complete replication both quantitatively and qualitatively for the potash comparison. As for the application of phosphate, each plot was divided into two sub-plots, only one of which, chosen at random, received superphosphate. The area of each sub-plot was 1/90 acre. Analysis showed that the response to potash dressings was the same for all the different qualities, and their yields have, therefore, for present purposes been averaged, yielding Table I below.

Table I. *Potatoes (Ally) Long Hoos, Section I*, 1929.

Average yield in tons per acre (y).

Cwt. of K_2O per acre	Without phosphate (cwt. of nitrogen per acre) 0	0·3	0·6	With phosphate (cwt. of nitrogen per acre) 0	0·3	0·6
0	4·52	5·10	5·38	4·79	5·52	5·94
0·5	4·86	5·26	5·41	5·01	5·90	6·28
1·0	4·62	5·30	5·59	4·89	5·77	6·51
Mean		5·12			5·62	
Reciprocal		0·19531			0·17793	

½ (difference) = 0·00869

Now, the measurement of agreement of the expected yields with those observed affords us a means of judging the accuracy of any hypothesis to be tested. It is therefore necessary to obtain values of the expected yields obeying the Resistance Formula and approximating to the observed.

Crude values of the reciprocals of the expected yields based on the Resistance Formula were first arrived at by averaging the yields obtained for each combination of N and K with and without phosphate, finding their reciprocals and adding to and subtracting from them half the difference of the reciprocals of the means of yields obtained with and without phosphate. This was done by means of the following table.

Table II.

Cwt. of K_2O per acre	(A) Average of the yield with and without phosphate (cwt. of nitrogen per acre) 0	0·3	0·6	(B) Reciprocals of (A) 0	0·3	0·6
0	4·66	5·31	5·66	0·21459	0·18832	0·17668
0·5	4·94	5·58	5·84	0·20243	0·17921	0·17123
1·0	4·76	5·54	6·05	0·21008	0·18010	0·16529

Table III. *Reciprocals of the expected yield* 1/m *built up by adding to and subtracting* 0·00869 *from the reciprocals in Table II (B).*

	Without phosphate			Mean (a)		With phosphate			Mean (a)	
	0·22328	0·19701	0·18537	0·20189		0·20590	0·17963	0·16799	0·18451	
	0·21112	0·18790	0·17992	0·19298		0·19374	0·17052	0·16253	0·17560	
	0·21877	0·18879	0·17398	0·19385		0·20139	0·17141	0·15660	0·17647	
Mean (b)	0·21772	0·19123	0·17976	0·19624 (c)	(b)	0·20034	0·17385	0·16237	0·17886	Mean (c)

Table IV (A). *Reciprocals of the expected yields built up from the margins of Table III by use of summation formula* a + b − c.

Without phosphate			With phosphate		
0·22337	0·19688	0·18541	0·20599	0·17950	0·16802
0·21446	0·18797	0·17650	0·19708	0·17059	0·15911
0·21535	0·18884	0·17735	0·19795	0·17146	0·15998

Table IV (B). *Values of first approximation of* m, i.e. *reciprocals of Table IV (A).*

Without phosphate			With phosphate		
4·477	5·079	5·393	4·854	5·571	5·951
4·662	5·320	5·666	5·074	5·862	6·285
4·643	5·296	5·638	5·052	5·832	6·251

For the purpose of improving these expectations we shall further need a table of m^4.

Table IV (C). *Values of* m^4.

	Without phosphate			Total	With phosphate			Total	Total
	402	666	846	1914	551	963	1254	2768	4682
	472	801	1030	2303	663	1181	1560	3404	5707
	465	787	1010	2262	651	1157	1526	3334	5596
Total	1339	2254	2886	6479	1865	3301	4340	9506	
	3204	5555	7226						15985

These fourth powers of the crude expectations are then used as weights in obtaining the improved fit, for they are inversely proportional to the variances of the reciprocals in the field trial.

Table V. *Reciprocals of the observed yields* (1/y) *from Table I.*

Without phosphate			With phosphate		
0·22124	0·19608	0·18587	0·20877	0·18116	0·16835
0·20576	0·19011	0·18484	0·19960	0·16949	0·15924
0·21645	0·18868	0·17889	0·20450	0·17331	0·15361

The margins in the following tables are built up by taking the weighted averages of the reciprocals of the observed yields, for rows and columns separately (with and without phosphate) and for the combined tables.

Table VI (A). *Weighted averages of the reciprocals of the observed yields.*

	Without phosphate			(a)		With phosphate			(a)	
				0·19685					0·18085	
				0·19096					0·17066	
				0·19002					0·17038	
(b)	0·21412	0·19137	0·18306	0·19237 (c)	(b)	0·20402	0·17423	0·15989	0·17353	(c)

Table VI (B).

				(a)	
				0·18739	
				0·17885	
				0·17832	
(b)	0·20824	0·18119	0·16914	0·18117	(c)

The difference of 0·18117 from 0·19237 represents the correction to be applied to the margins of Table VI (B) to get the margins of Table VII (A) below. Likewise the difference of 0·18117 from 0·17353 represents the correction to be applied to the table to get the margins of Table VII (B). The individual nine values of each set are then obtained from the margins by the relation $a + b - c$.

Table VII.

	(A) Without phosphate			(a)		(B) With phosphate			(a)	
	0·22566	0·19861	0·18658	0·19859		0·20682	0·17977	0·16772	0·17975	
	0·21712	0·19007	0·17802	0·19005		0·19828	0·17123	0·15918	0·17121	
	0·21659	0·18954	0·17749	0·18952		0·19775	0·17070	0·15865	0·17068	
(b)	0·21944	0·19239	0·18034	0·19237 (c)	(b)	0·20060	0·17355	0·16150	0·17353	(c)

The margins of Table VI (B) are further improved by applying various corrections which are found by taking the weighted average difference between the reciprocals of the observed and expected values of Tables V and VII respectively. The arithmetic is much facilitated by preparing the following table.

Table VIII. *Differences between the reciprocals of the observed and expected values.*

Without phosphate			With phosphate		
−0·00442	−0·00253	−0·00069	+0·00295	+0·00139	+0·00063
−0·01136	+0·00004	+0·00682	+0·00132	−0·00174	+0·00006
−0·00014	−0·00084	+0·00140	+0·00675	+0·00261	−0·00504

Table IX. *Product of differences and corresponding values of* m[4] *as obtained in Table IV (C).*

	Without phosphate			Total
	−1·77684	−1·68498	−0·58374	−4·04556
	−5·36192	+0·03204	+7·02464	+1·69472
	−0·06510	−0·66108	+1·41400	+0·68782
Total	−7·20386	−2·31402	+7·85486	−1·66302
	With phosphate			**Total**
	+1·62545	+1·33857	+0·79002	+3·75404
	+0·87516	−2·05494	+0·09360	−1·08618
	+4·39425	+3·01977	−7·69104	−0·27702
Total	+6·89486	+2·30340	−6·80742	+2·39084

Table X. *Corrections obtained by adding margins of Table IX and dividing by marginal* m[4] *in Table IV (C), with and without phosphate combined.*

			−0·00006
			+0·00011
			+0·00007
−0·00010	0	0·00014	−0·00026 for plots without phosphate +0·00025 for plots with phosphate

The value − 0·00026 is the quotient of − 1·66302 and 6479, while + 0·00025 is the quotient of + 2·39084 and 9506.

The improved values for the reciprocals of the expected yields after applying the corrections are shown in Table XI (A), *e.g.* 0·22524 (without phosphate) and 0·20691 (with phosphate) are obtained by adding − 0·00006, − 0·00010 and − 0·00026 to 0·22566 (without phosphate), and − 0·00006, − 0·00010 and + 0·00025 to 0·20682 (with phosphate) in Tables VII (A) and (B) respectively.

Table XI (A). *Reciprocals of the expected yields* (1/m).

Without phosphate			With phosphate		
0·22524	0·19829	0·18638	0·20691	0·17996	0·16805
0·21687	0·18992	0·17801	0·19854	0·17159	0·15968
0·21630	0·18955	0·17744	0·19797	0·17102	0·15911

Table XI (B). *Values of* m. *Reciprocals of values in Table XI (A).*

Without phosphate			With phosphate		
4·440	5·043	5·365	4·833	5·557	5·951
4·611	5·265	5·618	5·037	5·828	6·262
4·623	5·275	5·733	5·054	5·847	6·285

$$\overset{18}{\underset{1}{S}} (y - m)^2 = 0{\cdot}22927 \text{ (from Tables I and XI (B)).}$$

At this stage, two tests were applied to see how far the values of m as obtained in Table XI (B) supply a sufficient fit to the observed yields. If the process is sufficiently complete (i) $S(y-m)^2$ should be approximately at its minimum value; similarly (ii) $Sm^2(y-m)$, taken over all plots treated alike in respect to any one manure, must be small compared to the individual values of $m^2(y-m)$.

Table XI (C). $m^2(y-m)$ *from Tables I and XI* (*B*).

	Without phosphate			Total	With phosphate			Total
	+1·577	+1·449	+0·432	+3·458	−1·004	−0·842	−0·389	−2·235
	+5·294	−0·138	−6·565	−1·409	−0·685	+2·445	+0·706	+2·466
	−0·064	+0·695	−4·699	−4·068	−3·989	−2·632	+8·888	+2·267
Total	+6·807	+2·006	−10·832	−2·019	−5·678	−1·029	+9·205	+2·498

Sum of margins

			+1·223
			+1·057
			−1·801
+1·129	+0·977	−1·627	+0·479

The marginal totals are fairly high when compared with the individual entries, and thus indicate that the values of m as obtained above are not yet sufficiently fitted to the observed yields. Consequently an improved fit of the values of m is now sought by the "Method of Least Squares."

It is required to minimise the quantity $Q = S(y-m)^2$, where y stands for an observed yield and m may be expressed in terms of certain adjustable parameters which may be specified as follows:

$$1/m = N_p + K_q + P_r + C,$$

where P_r takes the two values P and O according as the plot does not or does receive phosphate;

K_q takes three values K, K' and O for none, single and double potash;

N_p takes three values N, N' and O for none, single and double nitrogen.

Then m is obtained from the six arbitrary parameters C, P, K, K', N, N'. The equations to be satisfied on minimising are

$$\frac{dQ}{dC} = \frac{dQ}{dP} = \frac{dQ}{dK} = \frac{dQ}{dK'} = \frac{dQ}{dN} = \frac{dQ}{dN'} = 0.$$

These are non-linear equations in six unknowns but, commencing with the approximate solution already obtained, the solution may be obtained by successive approximations.

Let C_0, P_0, K_0, K_0', N_0, N_0' be any set of approximate values and suppose $C = C_0 + c$, $P = P_0 + p$, etc., where c, p, k, k', n and n' are the corrections required. Then

$$\text{(i)}\quad 0 = \frac{dQ}{dC} = \frac{dQ}{dC_0} + c\frac{d^2Q}{dC_0^2} + p\frac{d^2Q}{dC_0 dP_0} + k\frac{d^2Q}{dC_0 dK_0} + k'\frac{d^2Q}{dC_0 dK_0'} + n\frac{d^2Q}{dC_0 dN_0} + n'\frac{d^2Q}{dC_0 dN_0'};$$

$$\text{(ii)}\quad 0 = \frac{dQ}{dP} = \frac{dQ}{dP_0} + p\frac{d^2Q}{dP_0^2} + c\frac{d^2Q}{dC_0 dP_0} + k\frac{d^2Q}{dP_0 dK_0} + k'\frac{d^2Q}{dP_0 dK_0'} + n\frac{d^2Q}{dP_0 dN_0} + n'\frac{d^2Q}{dP_0 dN_0'};$$

$$\text{(iii)}\quad 0 = \frac{dQ}{dK} = \frac{dQ}{dK_0} + k\frac{d^2Q}{dK_0^2} + k'\frac{d^2Q}{dK_0 dK_0'} + p\frac{d^2Q}{dK_0 dP_0} + c\frac{d^2Q}{dK_0 dC_0} + n\frac{d^2Q}{dK_0 dN_0} + n'\frac{d^2Q}{dK_0 dN_0'};$$

$$\text{(iv)}\quad 0 = \frac{dQ}{dK'} = \frac{dQ}{dK_0'} + k'\frac{d^2Q}{dK_0'^2} + k\frac{d^2Q}{dK_0 dK_0'} + p\frac{d^2Q}{dK_0' dP_0} + c\frac{d^2Q}{dK_0' dC_0} + n\frac{d^2Q}{dK_0' dN_0} + n'\frac{d^2Q}{dK_0' dN_0'};$$

$$\text{(v)}\quad 0 = \frac{dQ}{dN} = \frac{dQ}{dN_0} + n\frac{d^2Q}{dN_0^2} + n'\frac{d^2Q}{dN_0 dN_0'} + p\frac{d^2Q}{dN_0 dP_0} + c\frac{d^2Q}{dN_0 dC_0} + k\frac{d^2Q}{dN_0 dK_0} + k'\frac{d^2Q}{dN_0 dK_0'};$$

$$\text{(vi)}\quad 0 = \frac{dQ}{dN'} = \frac{dQ}{dN_0'} + n'\frac{d^2Q}{dN_0'^2} + n\frac{d^2Q}{dN_0 dN_0'} + p\frac{d^2Q}{dN_0' dP_0} + c\frac{d^2Q}{dN_0' dC_0} + k\frac{d^2Q}{dN_0' dK_0} + k'\frac{d^2Q}{dN_0' dK_0'}.$$

Thus we have six equations which are linear in c, p, k, k', n and n' with known coefficients, for

$$\frac{dQ}{dC_0} = -2S(y-m)\frac{dm}{dC_0} = 2\overset{18}{\underset{1}{S}}\{m^3(y-m)\},$$ the summation being over all the eighteen plots;

$$\frac{dQ}{dP_0} = 2\overset{9}{\underset{1}{S}}m^2(y-m)$$ over plots without phosphate;

$$\frac{dQ}{dK_0} = 2\overset{6}{\underset{1}{S}}m^2(y-m)$$ over plots without potash;

$$\frac{dQ}{dK_0'} = 2\overset{6}{\underset{1}{S}}m^2(y-m)$$ over single potash plots;

$$\frac{dQ}{dN_0} = 2\overset{6}{\underset{1}{S}} m^2 (y - m) \qquad \text{over plots without nitrogen;}$$

$$\frac{dQ}{dN_0'} = 2\overset{6}{\underset{1}{S}} m^3 (y - m) \qquad \text{over single nitrogen plots.}$$

These quantities can all be obtained readily from Table XI (C).

$$\frac{d^2Q}{dC_0^2} = 2\frac{d}{dC_0}\overset{18}{\underset{1}{S}} m^2 (y - m) = 2\overset{18}{\underset{1}{S}} \{m^4 - 2m^3 (y - m)\} \quad \text{summation being over all the eighteen plots;}$$

$$\frac{d^2Q}{dC_0 dP_0} = 2\overset{9}{\underset{1}{S}} \{m^4 - 2m^3 (y - m)\} \qquad \text{over plots without phosphate;}$$

$$\frac{d^2Q}{dP_0 dK_0'} = 2\overset{3}{\underset{1}{S}} \{m^4 - 2m^3 (y - m)\} \qquad \text{over plots without phosphate but with single dressing of potash;}$$

$$\frac{d^2Q}{dC_0 dK_0} = 2\overset{6}{\underset{1}{S}} \{m^4 - 2m^3 (y - m)\} \qquad \text{over plots without potash;}$$

and so on

$$\frac{d^2Q}{dK_0 dK_0'} = \frac{d^2Q}{dN_0 dN_0'} = 0.$$

The values of $\frac{d^2Q}{dC_0^2}$, $\frac{d^2Q}{dC_0 dP_0}$, etc., are obtained from the following table:

Table XII. $m^4 - 2m^3 (y - m)$.

	Without phosphate			Total	With phosphate			Total
	374·637	632·166	823·828	1830·631	555·304	962·938	1258·787	2777·029
	403·209	769·857	1069·923	2242·989	650·589	1125·184	1528·819	3304·592
	457·355	766·947	1134·130	2358·432	692·766	1199·534	1448·610	3340·910
Total	1235·201	2168·970	3027·881	6432·052	1898·659	3287·656	4236·216	9422·531

Sum of margins

			4607·660
			5547·581
			5699·342
3133·860	5456·626	7264·097	15854·583

The values of $\frac{dQ}{dC_0}$, $\frac{d^2Q}{dC_0^2}$, $\frac{d^2Q}{dC_0 dP_0}$, ..., etc. when substituted in the six equations enable them to be solved for c, p, k, k', n and n', with the following results:

$$c = -0{\cdot}000258; \qquad k' = +0{\cdot}000028;$$
$$p = +0{\cdot}000158; \qquad n = -0{\cdot}000160;$$
$$k = -0{\cdot}000045; \qquad n' = +0{\cdot}000022.$$

The table of corrections is then built up from the relation

$$c + x + y + z,$$

where x takes the values p and o according as it is applied to plots without phosphate and plots with phosphate respectively. Similarly y takes the values k, k' and o for none, single and double potash; z takes the values n, n' and o for none, single and double nitrogen.

Table XIII. *Table of corrections.*

Without phosphate (0·000)			With phosphate		
−31	−12	−15	−46	−28	−30
−23	−05	−07	−39	−21	−23
−26	−08	−10	−42	−24	−26

Table XIV (A). *Corrected values of the reciprocals of the expected yields by adding values in Tables XI (A) and XIII.*

Without phosphate			With phosphate		
0·22493	0·19817	0·18623	0·20645	0·17968	0·16775
0·21664	0·18987	0·17794	0·19815	0·17138	0·15945
0·21604	0·18927	0·17734	0·19755	0·17078	0·15885

Table XIV (B). *Values of* m *reciprocals of Table XIV (A).*

Without phosphate			With phosphate		
4·446	5·046	5·370	4·844	5·565	5·961
4·616	5·267	5·620	5·047	5·835	6·271
4·629	5·283	5·639	5·062	5·855	6·295

$$\overset{18}{\underset{1}{S}} (y - m)^2 = 0{\cdot}20899.$$

Table XIV (C). $m^2 (y - m)$ *from Tables I and XIV (B).*

	Without phosphate			Total	With phosphate			Total
	+1·466	+1·371	+0·297	+3·134	−1·262	−1·408	−0·755	−3·425
	+5·200	−0·187	−6·944	−1·931	−0·934	+2·213	+0·332	+1·611
	−0·188	+0·462	−1·555	−1·281	−4·408	−2·931	+8·511	+1·172
Total	+6·478	+1·646	−8·202	−0·078	−6·604	−2·126	+8·088	−0·642

Combined marginal totals

			−0·291
			−0·320
			−0·109
−0·126	−0·480	−0·114	−0·720

The marginal totals are now much reduced, indicating that the value of m has been further improved.

Test of Goodness of Fit.

So far, we have obtained the theoretical values based on the Resistance Formula. It now remains to see whether these values are in agreement with the values obtained by the experiment. The method known as the analysis of variance developed by Dr R. A. Fisher (2) is applied to test the validity of the hypothesis. From the analysis of variance for this experiment it is found that after apportioning fractions of the total sum of squares of deviations of the yields from their general mean to various known causal factors, the following residual amounts are left for (i) the N and K error, and (ii) the phosphate error. (For an explanation of a similar analysis but with only three blocks see Wishart and Clapham (3)).

Analysis of variance due to:	Degrees of freedom	Sum of squares	Mean square
(i) Non-adjacent sub-plots (N and K error)...	60	231278·18	3854·64
(ii) Adjacent sub-plots (phosphate error) ...	60	59271·89	987·86

These figures are based on the yields in ¼ lb. units per 1/90th of an acre. Expressed in tons per acre, the mean squares for non-adjacent and adjacent sub-plots are 0·38800 and 0·09968 respectively. But the yields in our tables are based on the means of nine sub-plots; so to make the figures comparable we have to divide them by nine. Thus

Analysis of variance due to:	Degrees of freedom	Sum of squares	Mean square
(i) Non-adjacent sub-plots (N and K error)...	60	2·5928	0·04321
(ii) Adjacent sub-plots (phosphate error) ...	60	0·6648	0·01108

There are seventeen degrees of freedom for the eighteen values of the table out of which five have been accounted for in fitting the constants, leaving twelve degrees of freedom in which the observed values might differ from expectations. The twelve degrees of freedom can further be split up into (i) eight for differences from expectation due to fertility of adjacent sub-plots, and (ii) four for the non-adjacent ones. The total sum of the squares of the deviations of the expected yields from the observed ones is

$$\overset{18}{\underset{1}{S}} (y - m)^2 = 0{\cdot}20899.$$

The sum of the squares of the deviations of the expected yields from the observed values for eight degrees of freedom due to fertility of adjacent sub-plots was calculated from the following tables.

Table XV. $(y - m)$ *from Tables I and XIV (B).*

Without phosphate			With phosphate		
+0·074	+0·054	+0·010	−0·054	−0·045	−0·021
+0·244	−0·007	−0·210	−0·037	+0·065	+0·009
−0·009	+0·017	−0·049	−0·172	−0·085	+0·215

Table XVI. *Differences, phosphate—no phosphate, of* (y − m) *from Table XV.*

−0·128	−0·099	−0·031
−0·281	+0·072	+0·219
−0·163	−0·102	+0·264

½ (sum of squares of the above nine figures) = 0·13270.

The difference of 0·13270 from 0·20899 represents the sum of the squares of the deviations of the expected yields from the observed values with four degrees of freedom for the non-adjacent plots.

If the Resistance Formula has been satisfactorily fitted, the mean sum of squares with the eight degrees of freedom should not be significantly larger than that obtained from the field error for the phosphate comparison; similarly, the mean square of deviations with four degrees of freedom should not be significantly larger than that obtained for the N and K comparison in the field experiment. We have:

Analysis of variance due to:	Degrees of freedom	Sum of squares	Mean square	½ $\log_e$ (mean square + 100)
(i) Resistance formula	4	0·0763	0·01907	0·3227
Non-adjacent sub-plots (N and K error)	60	2·5928	0·04321	0·7317

In this case the mean square from the Resistance Formula is actually less than that for the corresponding field error, $z = -0{\cdot}4090$.

Analysis of variance due to:	Degrees of freedom	Sum of squares	Mean square	½ $\log_e$ (mean square + 100)
(ii) Resistance formula	8	0·1327	0·01659	0·2531
Adjacent sub-plots (P error) ...	60	0·6648	0·01108	0·0512

In this case, although the mean square of deviations for adjacent sub-plots is greater than the mean square for the corresponding field error, it is not significant, as is shown by the value of z. To reach the 5 per cent. point the value of z is required to be 0·3702. It is evident that the Resistance Formula has fitted the data satisfactorily in this case.

III. Constants of the Resistance Formula.

Bh. Balmukand has shown in the data examined by him that $F(N)$, $F'(K)$, etc. can be taken to be of the form $\frac{a_n}{n + N}$ and $\frac{a_k}{k + K}$, where a_n and a_k are the constants varying with the nutrient and the crop, while n and k represent the available nitrogen and potash in the unmanured soil. The fitness of this special formula cannot be tested from our data, but the values of the constants n, a_n, k, a_k are calculated by means of this formula and their standard errors determined by the Method of Maximum Likelihood as developed by him. In view of the fact that there are only two levels of phosphate the values for the constants p and a_p cannot be determined. From the Table XIV (A) we find that the difference between the reciprocals of the expected yields for no nitrogen and 0·3 cwt. of nitrogen is practically 0·02677—any departure from constancy in the last place being really due to the neglect of the sixth decimal place—and that between those of 0·3 and 0·6 of nitrogen is 0·01193. Similarly for potash we have 0·0083 and 0·0006.

We have the following equations:

$$\text{(i)} \qquad \frac{a_n}{n} - \frac{a_n}{n+0{\cdot}3} = 0{\cdot}02677;$$

$$\text{(ii)} \quad \frac{a_n}{n+0{\cdot}3} - \frac{a_n}{n+0{\cdot}6} = 0{\cdot}01193.$$

Solving, we find

$n = 0{\cdot}4823$ cwt. of N_2 = 2·305 cwt. of sulphate of ammonia per acre;

$a_n = 0{\cdot}0337$ cwt. of N_2 = 0·161 cwt. of sulphate of ammonia per ton of potato.

(1 cwt. of nitrogen is contained in 4·78 cwt. of sulphate of ammonia.)

Similarly

$$\text{(i)} \qquad \frac{a_k}{k} - \frac{a_k}{k+0{\cdot}5} = 0{\cdot}0083;$$

$$\text{(ii)} \quad \frac{a_k}{k+0{\cdot}5} - \frac{a_k}{k+1} = 0{\cdot}0006.$$

On solving, we have

$k = 0{\cdot}0779$ cwt. of K_2O = 0·157 cwt. of sulphate of potash per acre;

$a_k = 0{\cdot}0007$ cwt. of K_2O = 0·0015 cwt. of sulphate of potash per ton of potato.

(1 cwt. of K_2O is contained in 2·02 cwt. of sulphate of potash.)

Standard errors of the constants.

The calculation of the standard errors was as follows:

Table XVII (A). *Values of* m⁴ *as obtained from Table XV (B).*

	Without phosphate			Total	With phosphate			Total
	390·671	648·405	831·382	1870·458	550·480	959·401	1262·843	2772·724
	453·988	769·452	997·480	2220·920	648·670	1159·205	1540·456	3348·331
	461·559	779·242	1011·056	2251·857	656·584	1175·585	1570·553	3402·722
Total	1306·218	2197·099	2839·918	6343·235	1855·734	3294·191	4373·852	9523·777

Combined marginal totals

				w_{kt}
				4643·182
				5569·251
				5654·579
w_{nt}	3161·952	5491·290	7213·770	15867·012

Table XVII (B).

	w_{nt}	$n+N_t$ (cwt. of sulphate of ammonia)	$\frac{w_{nt}}{n+N_t}$	$\frac{w_{nt}}{(n+N_t)^2}$	$\frac{w_{nt}}{(n+N_t)^3}$	$\frac{w_{nt}}{(n+N_t)^4}$
1	3161·952	2·305	1371·78	595·13	258·19	112·01
2	5491·290	3·739	1468·65	392·79	105·05	28·10
3	7213·770	5·173	1394·50	269·57	52·11	10·77
Total	15867·012	—	4234·93	1257·49	415·35	150·88

Table XVII (C).

t	w_{kt}	$k+K_t$ (cwt. of sulphate of potash)	$\frac{w_{kt}}{k+K_t}$	$\frac{w_{kt}}{(k+K_t)^2}$	$\frac{w_{kt}}{(k+K_t)^3}$	$\frac{w_{kt}}{(k+K_t)^4}$
1	4643·182	0·157	29574·41	188372·02	1199821·80	7642177·11
2	5569·251	1·167	4772·28	4089·36	3504·16	3002·71
3	5654·579	2·177	2597·42	1193·12	548·06	251·75
Total	15867·012	—	36944·11	193654·50	1203874·02	7645431·57

The values of the standard errors were obtained by using the formula given by Bh. Balmukand.

$$V(a_n) = \sigma^2 \frac{\begin{vmatrix} \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^4} & -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} \\ -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} & \overset{3}{\underset{1}{S}} w_{nt} \end{vmatrix}}{\begin{vmatrix} \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} & -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^3} & \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)} \\ -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^3} & \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^4} & -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} \\ -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)} & -\overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} & \overset{3}{\underset{1}{S}} w_{nt} \end{vmatrix}},$$

$$V(n) = \frac{\sigma^2}{(a_n)^2} \begin{vmatrix} \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)^2} & \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)} \\ \overset{3}{\underset{1}{S}} \frac{w_{nt}}{(n+N_t)} & \overset{3}{\underset{1}{S}} w_{nt} \end{vmatrix} \div D,$$

where D is the denominator of the previous fraction.

Substituting the respective values:

$\sigma_n = 0·1158$ cwt. of sulphate of ammonia,

$\sigma_{a_n} = 0·1182$ cwt. of sulphate of ammonia.

Similarly:

$\sigma_{(k)} = 1·1552$ cwt. of sulphate of potash,

$\sigma_{a_k} = 0·0122$ cwt. of sulphate of potash.

Expressing the constants with their standard errors in lb. of nitrogen and K_2O, we have:

$$n = 54{\cdot}02,\ \text{S.E. } 2{\cdot}71, \qquad k = 8{\cdot}70,\ \text{S.E. } 64{\cdot}05,$$
$$a_n = 3{\cdot}77,\ \text{S.E. } 2{\cdot}77, \qquad a_k = 0{\cdot}08,\ \text{S.E. } 0{\cdot}68.$$

With the exception of n, for which the standard error is only 5 per cent., the standard errors we have obtained for the constants of the Resistance Formula are very high. The values of the constants together with their standard errors are set out in the following table and the results previously obtained by Bh. Balmukand, converted into pounds of nitrogen and K_2O respectively, are added for comparison.

Table XVIII.

	n	S.E.	a_n	S.E.	k	S.E.	a_k	S.E.
Long Hoos (Ally) 1929, dung (14 tons)	54·02	2·71	3·77	2·77	8·70	64·05	0·08	0·68
Stackyard (Kerr's Pink), 1926 ...	40·71	19·91	2·31	1·44	26·61	33·27	0·49	0·71
Seale Hayne: dung (10 tons)	38·89	9·37	3·96	1·12	62·65	32·16	1·43	0·94
Seale Hayne: undunged, 1927 ...	13·35	5·15	1·85	0·80	−5·55	4·99	0·42	0·35

IV. SUMMARY AND CONCLUSION.

The table of results given in the last section shows that for all experiments for which the Resistance Formula has been fitted the values of the constants are consistent, bearing in mind the standard errors to which these values are subject. The constants for nitrogen may be held to have been determined with some approach to precision, but those for potash are not so well determined. We find at Rothamsted, for example, that the crop responds to a moderate dressing of potash, but higher dressings do not usually improve the yields any further. This fact limits the precision of the formula. The reader is asked to refer at this point to the full descriptions as to the meaning to be attached to the constants of the formula given by Bh. Balmukand in the earlier paper (1). The constants a_n and a_k, called the importance factors, are interpreted as determining the capacity of the crop to recover the particular nutrient out of the soil. The chemical analysis of the tubers grown under nitrogen and potash starvation conditions should, therefore, furnish us with values which should be of the same order of magnitude as obtained by the formula. The minimum nitrogen percentage figure is 0·204, which when reduced to our units is equivalent to 4·57 lb. of nitrogen per ton of potato. The values obtained from the Resistance Formula are as close as the standard errors allow us to expect, and are also of the same order of magnitude as the chemical composition of the tubers demands.

The 1929 potato experiment studied in this paper may be regarded as confirming the conclusions of Balmukand's paper as to the possibility of fitting the Resistance Formula to experimental data. It has confirmed the values previously reached for the nitrogen constants on a dunged soil, but the corresponding constants for potash cannot yet be regarded as well determined from any experiment so far examined. It so happens that only a moderate response to potash in the case of potatoes occurs on the Rothamsted soil, and this has led in the past few years to a reduction in the quantity applied in order to have a number of suitable levels. This policy has not yet succeeded, and one may anticipate that the unit dressing will be still further reduced in future. The precision of the Resistance Formula depends upon a suitable number of such levels being incorporated into any experiment, at each of which some response is shown to the dressing. So we can only hope to improve the precision of the determination of the potash constants by still further reducing the unit dressing. It is hoped that an experiment on these lines now being initiated at Rothamsted will furnish the information sought.

Finally it is a pleasure to record my indebtedness to Dr R. A. Fisher, who suggested the problem of this paper and whose help at various stages has been invaluable, and also to Dr J. Wishart of the Rothamsted Experimental Station for much valuable advice and criticism.

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