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Restricted randomization for blocks of sixteen plots

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Grundy & Healy (1950) developed a method of 'restricted randomization' applicable to factorial experiments of 2^n or 3^n type laid out either in quasi-Latin squares of size 8×8 or 9×9, or in blocks of 8 or 9 plots. This method of randomization, while in no way affecting the validity of the normal analysis, prevents the appearance of designs with obviously undesirable features. Grundy & Healy gave an example of a quasi-Latin square in which the main effect of one factor corresponded to the pattern:

```
− − − − + + + +
− − − − + + + +
− − − − + + + +
− − − − + + + +
+ + + + − − − −
+ + + + − − − −
+ + + + − − − −
+ + + + − − − −
```

In such an arrangement the error mean square is likely to be an underestimate of the variance applicable to the treatment means. Under full randomization such arrangements are balanced in the long run by others with the opposite characteristic in which the error mean square is an overestimate of the treatment variance.

The method of restricted randomization avoids extreme designs of both types and so eliminates the dilemma of the experimenter who by random choice arrives at a 'bad' randomization; to accept this or to tear it up and start again?

Recent series of experiments at Rothamsted and elsewhere have involved blocks of 16 plots; for example a series on the effects of green manures tested two factors each at 4 levels and confounding seemed undesirable—see Dyke (1962).

Grundy and I therefore set about finding sets of contrasts between 16 plots (corresponding to a factor at 2 levels) with the following properties (Grundy & Healy, 1950):

(i) Each set contains 15 contrasts which with the unit element

++++ ++++ ++++ ++++

form a group under 'multiplication'. (I separate the signs into groups of 4 for convenience later in the discussion.)

(ii) No contrast contains more than 4 consecutive like signs.

To enumerate all possible sets seemed excessively laborious and Grundy suggested and made a start on a limited range of sets which I have completed since his death.

This involves the interlacing of contrasts taken from the sets of 8 given by Grundy and Healy. Thus the signs in places 1, 3, 5, 7, 9, 11, 13, 15 form one of these contrasts and the signs in places 2, 4, 6, 8, 10, 12, 14, 16 form another.

Now I define five types of contrast:

Type α has in the first 8 places 4 + signs and 4 − signs.

Type β has in the first 8 places 5 + signs and 3 − signs (or 3 and 5).

Type γ has in the first 8 places 6 + signs and 2 − signs (or 2 and 6).

Type δ has in the first 8 places 7 + signs and 1 − sign (or 1 and 7).

Type ϵ has in the first 8 places 8 + signs or 8 − signs.

The process used has yielded a total of 83 distinct sets of 15 contrasts satisfying conditions (i) and (ii) and a further condition:

(iii) Just 2 of the contrasts of each set are of type γ, the rest of types α and β.

(A few sets were found with more than 2 contrasts of type γ; these are not further considered.)

I now show that no set of 15 contrasts of types α, β and γ can contain fewer than 2 of type γ, so that a fuller enumeration would not lead to 'better' sets than those already obtained.

(I) The rules of multiplication are

$\alpha . \alpha = \alpha$ or γ (I exclude δ and ϵ contrasts),
$\beta . \beta = \alpha$ or γ,
$\beta . \alpha = \beta$.

(II) Any set of 15 contrasts must contain 1, 3, 7 or 15 'even' contrasts (i.e. α's, γ's or ϵ's). But there are only 7 orthogonal contrasts between 8 objects so the case of 15 'even' contrasts must comprise at best 7 α's and 8 γ's or ϵ's. Hence any acceptable set must comprise 7 'even' contrasts and 8 β's.

(III) Consider any β of the set. Suppose it has 5 + signs and 3 − signs in the first 8 places. Consider the signs in other contrasts which correspond to the 3 − signs of the β. Multiplying any contrast by the β will change the signs in these three places.

There are 8 possibilities for the 'even' contrasts:

$$\begin{matrix} ++- & +-+ & +-- & -++ \\ -+- & --+ & +++ & --- \end{matrix}$$

If the 'even' contrasts are all α's then neither $+++$ nor $---$ is admissible because their products with β are δ contrasts. Hence there are not more than 6 α's in an acceptable set.

(IV) Now consider a set containing 6 α's and 1 γ. Among the 6 α's there are relations of the type

$$\alpha_1 . \alpha_2 = \alpha_3.$$

Now suppose the γ has 2 − signs in the first 8 places. In these places all the α's must have + − or − +, because $\alpha . \gamma$ must be an α. Hence no such set exists.

A key to the 83 sets satisfying conditions (i), (ii) and (iii) is given below. Each set is defined by 3 contrasts; the remainder are obtained by successive multiplication of these and the contrast + − + − + − + − + − + − + − + −, which belongs to every set.

Key to sets of contrasts

Code for groups of 4 consecutive signs:

+ + + + : A	+ + + − : B	+ + − + : C	+ + − − : D
+ − + + : E	+ − + − : F	+ − − + : G	+ − − − : H
− + + + : I	− + + − : J	− + − + : K	− + − − : L
− − + + : M	− − + − : N	− − − + : O	− − − − : P

Defining contrasts of the 83 sets are given in Table 1. The sets are numbered to help in making a random choice. Where two numbers are entered opposite one set the first represents the set as given, the second the same set with each contrast written down in reverse order; sets with a single number are not altered by being reversed.

Table 1

1, 2	CPEJ	CENO	DHKE	3, 4	CFML	COFM	BHCO
5, 6	CPEJ	CGPE	BHCO	7, 8	CPEJ	CGPE	DNCG
9, 10	DBHK	CGPE	DKBN	11, 12	DBHK	CGPE	CPBJ
13, 14	CBGP	DHKE	CPBJ	15, 16	BILN	ALMH	DNCG
17, 18	BILN	ALMH	DHKE	19, 20	DKBN	ALMH	BHCO
21, 22	FOIG	CFMO	DCPE	23, 24	HEIO	CPEM	DCPE
25, 26	AJLN	CGPE	DNIG	27, 28	AJLN	CGPE	BHIO
29, 30	ALDH	CGPE	BHIO	31, 32	FMOI	GBHK	BCNO
33, 34	FOGC	EDNK	DCPE	35, 36	DECP	CGPE	AJLN
37, 38	DBGO	CGPE	AJLN	39, 40	BDHK	BOMD	COHE
41, 42	BDHK	ALMH	COHE	43, 44	BDHK	ALMH	CPDB
45, 46	BHCO	ALMH	DLHA	47, 48	EDHK	ANOM	HEOC
49, 50	EDHK	CFOG	FOMC	51, 52	ELNC	CPGE	FMOI
53, 54	ELNC	CFOG	FMOI	55, 56	GDNI	CPGE	HEOC
57, 58	DHIM	DKGF	BLNC	59, 60	DNAO	DAOH	BDHK
61, 62	BHCO	DIEP	BMLJ	63, 64	DNAO	BCGP	DELD
65, 66	DNAO	BION	DGJJ	67, 68	BHCO	DCMN	BOJD
69	FOMC	CPEM	DCPE	70	FMOI	CPEM	DIHG
71	FMOI	CFMO	DCPE	72	HECO	BNLI	DCPE
73	HECO	EJLM	DCPE	74	HECO	EJLM	GGPA
75	HECO	ANOM	DHLA	76	HECO	ANOM	GDLE
77	HECO	FJOI	DHLA	78	EDHK	FILM	HEOC
79	EDHK	HALG	FOMC	80	ELNC	FILM	HGMI
81	ELNC	HKDE	FMOI	82	ELNC	HALG	FMOI
83	GDNI	HKDE	FOMC				

Table 2

1	+ − + +	− + − −	− − + −	+ + − +	(ELNC reversed)
2	+ − − +	+ − − −	− + − +	+ − + +	(CFOG reversed)
3	+ + + −	+ − − −	+ + − −	− + − +	(FMOI reversed)
4	− + − +	− + − +	− + − +	− + − +	(FFFF reversed)
5	+ + − +	− − + +	+ − − −	+ − − +	(= 1.2)
6	+ − + −	− − + +	− − − +	− + + +	(= 1.3)
7	− − − +	+ + + −	+ − − −	− + + +	(= 1.4)
8	− − + +	− − + −	+ + + +	− − − +	(= 2.4)
9	− + − −	− − + −	− + + −	+ + + +	(= 3.4)
10	+ + − −	− + − −	+ − + +	− − + +	(= 1.2.3)
11	− + + +	+ − − +	− − + −	− − + +	(= 1.2.4)
12	− − − −	+ − − +	+ − + +	+ + − +	(= 1.3.4)
13	− − + −	− + − +	+ + − −	+ − + +	(= 2.3.4)
14	− + + −	+ + + −	− − − +	+ − − +	(= 1.2.3.4)
15	+ − − −	+ + + +	− + + −	− − − +	(= 2.3)

Example

Random choice of set—say 54 (ELNC, CFOG, FMOI in reverse order). The 15 contrasts are shown in Table 2. Assign the first factor to a contrast chosen from 1 to 15, say 6. Assign the second factor to any contrast other than 6, say 3. Now $3.6 = 3.1.3 = 1$. So we assign the third factor to any contrast except 1, 3 or 6—say 13 ($= 2.3.4$). The interactions of the factors already assigned includes contrasts 14 ($= 1.2.3.4$), 8 ($= 2.4$) and 11 ($= 1.2.4$). Hence the fourth factor can be assigned to any contrast except 1, 3, 6, 8, 11, 13, or 14. The process is completed by allocating the presence or higher level of each factor to + or − at random. If there are more than 4 factors their arrangement is obtained by considering the scheme of confounding required.

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