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Miscellanea

Residuals of Equal Magnitude in Fractional Replicate 2^n Designs

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SUMMARY

In half-replicate, quarter-replicate, etc., 2^n experiments, with error estimated from the highest order unaliased interactions, with $n = 4p$ (p an integer) the residuals occur in pairs of equal magnitude and like sign; with $n = 4p+2$, in pairs of equal magnitude and opposite sign.

Keywords: RESIDUAL; MAIN EFFECT; INTERACTION; FRACTIONAL REPLICATE

I RECENTLY examined the analysis of the results of a field experiment in which the plot-residuals occurred in pairs of equal magnitude and like sign. This is in contrast to the occurrence, more often noticed, of pairs of residuals of equal magnitude and opposite sign. To explain the anomaly, I start with an example.

Consider a 2^4 experiment with factors A, B, C, D, laid down as half-replicate with $I \equiv ABCD$, in one block of eight plots, the chosen half being the one that includes the untreated plot denoted by the symbol (1). The Yates table (Yates, 1937) of main effects and interactions is:

	Treatment								
Contrast	(1)	*ab*	*ac*	*bc*	*ad*	*bd*	*cd*	*abcd*	(*alias*)
A	−	+	+	−	+	−	−	+	(*BCD*)
B	−	+	−	+	−	+	−	+	(*ACD*)
C	−	−	+	+	−	−	+	+	(*ABD*)
D	−	−	−	−	+	+	+	+	(*ABC*)
AB	+	+	−	−	−	−	+	+	(*CD*)
AC	+	−	+	−	−	+	−	+	(*BD*)
AD	+	−	−	+	+	−	−	+	(*BC*)

Each row defines a contrast of four plots *versus* four plots. Suppose each to be calculated as a difference between mean yields. The yield of any one plot can be expressed in terms of the grand mean and the seven contrasts by reading the signs in the appropriate column. For example the yield of the untreated plot (1) = mean $-\frac{1}{2}A\ -\frac{1}{2}B\ -\frac{1}{2}C\ -\frac{1}{2}D\ +\frac{1}{2}AB\ +\frac{1}{2}AC\ +\frac{1}{2}AD$.

In this oversimple example suppose the unaliased two-factor interactions are used to estimate "error" i.e. residual variation. More realistic cases with more plots and more factors will be mentioned later. Then, in the above expression for the yield of the untreated plot, the first four terms define the fitted value and the last three constitute the error component or residual.

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Inspection of the Yates table shows that plots (1) and *abcd* both have + signs against the contrasts *AB*, *AC*, *AD*, used to estimate error. Hence the residuals of these two plots are equal (and of like sign).

Similarly for any pair of treatments such that, between them, they involve the presence of each factor exactly once: *ab* with *cd*, *ac* with *bd* and so on.

If the experiment is a $\frac{1}{4}$, $\frac{1}{8}$, etc. replicate certain rows and columns of the Yates table are omitted but provided that the four-factor interaction *ABCD* remains one of the defining identities, so that pairs of complementary treatments (e.g. *ab* and *cd*) remain, the above still holds good.

When the number of factors, n, varies, three cases emerge.

(i) and (ii) If n is even; suppose $n = 2p$. Then interactions of more than p factors are all aliased with simpler interactions or main effects. One half of the interactions of p factors are aliased with the remainder.

Then (i) if p is even the signs in columns paired as above are like, (ii) if p is odd they are unlike.

(iii) If n is odd, and the identities include the all-factor interaction, complementary pairs of treatments do not occur, and residuals are not paired. It is possible, nevertheless, to find designs and to select contrasts for estimating error, so that paired residuals do occur but such examples are rare.

Reference

Yates, F. (1937). The design and analysis of factorial experiments. Commonwealth Bureau of Soils, Harpenden, Technical Communications No. 35.