British Journal of Applied Physics

SPECIAL ARTICLE

The role of vegetation in meteorology, soil mechanics and hydrology

To cite this article: H L Penman 1951 *Br. J. Appl. Phys.* **2** 145

View the article online for updates and enhancements.

SPECIAL ARTICLE

The role of vegetation in meteorology, soil mechanics and hydrology*

By H. L. PENMAN, M.Sc., Ph.D., F.Inst.P., Rothamsted Experimental Station, Harpenden, Herts

Plants transpire large quantities of water at a rate primarily dependent upon weather conditions, and calculable from easily measured weather elements. They therefore play a dominant part in the water and heat balances of the earth's surface, and affect land drainage, underground storage and river flow. Calculation of the transpiration rate can be made the basis of controlled irrigation to produce maximum growth without waste of water, and it is shown that the principle can be applied equally successfully to a field, to a catchment area and to the British Isles as a whole. When the plants grow in a clay soil, shrinkage may occur as the soil dries, with damaging results on roads or buildings carried on the soil.

In a previous survey[1] the dependence of natural evaporation on weather was examined in detail, but, because of concentration on the mathematics and physics of the problem, it was not possible to give much attention to the practical application of the results. The present survey will remedy this defect. The basic principle is simple and will be outlined in the next section without giving details of the theories and experiments on which it is based. It amounts to answering "What happens to the sunshine?" and its simplicity arises from the close interdependence of two primary needs of the plant world—a supply of water and a supply of radiant energy. When plants get both they make very inefficient use of them, for most of the energy is used in evaporating nearly all of the water, the amounts taking part in photosynthesis being absurdly small. To the physicist this is a fortunate circumstance because it makes possible the drawing up of an energy balance sheet in a way that leaves evaporation as the only unknown, i.e. natural evaporation can be estimated from suitable weather measurements.

WHAT HAPPENS TO THE SUNSHINE?

The three main components of incoming solar radiation are indicated in the top left corner of Fig. 1, and below, the day regime is represented approximately to scale. After a loss by reflexion, the remaining energy is degraded in various ways. There is a complex interchange of long wave radiation, upward from the earth and downward from the clouds and the water vapour in the atmosphere: only the net outflow is represented in Fig. 1. There are two sinks for sensible heat, one in the soil, and the other in the air which is warmed by turbulent transfer of heat from the ground. Vegetation provides two sinks, a weak sink in taking up energy for photosynthesis and a very much stronger sink where the energy is consumed as latent heat of vaporization. By night, conditions are simpler, for there are only three components. Back radiation is maintained and this must draw its energy from the soil and the air. For a 24 h balance the day and night balances must be combined,

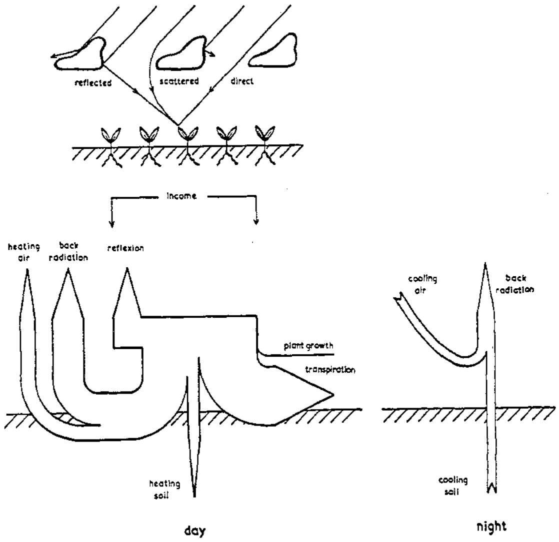

Fig. 1. Energy balance for average summer conditions

and as the air and soil components change sign between day and night their net effect is small. Table 1 gives

Table 1. *Energy balance May to September, 1949*

Energy sink	*Evaporation equivalent (in)*	%
Plant growth	0·4	1
Heating soil (*S*)	0·7	2
Heating air (*K*)	1·4	4
Reflexion (*r*)	7·2	20
Back radiation (R_B)	12·3	34
Transpiration (*E*)	14·0	39
Total (R_C)	36·0	100

a picture of average conditions over four months in the summer of 1949, the right-hand column showing the fraction of incoming energy taken up by each sink. The second column gives the evaporation equivalent for the whole period: taking the latent heat of vaporization as 590 cal/g, the evaporation equivalent is 1 in evaporated = 1 500 cal/cm². (The total income represents about 2½ million kWh/acre.) Accepting these figures as being correct in order of magnitude, the first two sinks can be

* Based on a lecture given on 20 October, 1950, to the Manchester Branch of The Institute of Physics.

ignored in most attempts to estimate evaporation from energy balance: heating of the air requires about one-tenth of the energy needed for evaporation, so it must be kept in.

The problem then is to evaluate incoming solar radiation, reflexion, back radiation, and heating of the air. The first is not difficult, as suitable radiation recorders exist, but their number is limited. Fortunately there is a very close relation between radiation and duration of bright sunshine, and in the absence of marked bias in the distribution of sunshine during the day the incoming intensity can be estimated for periods of a week or longer (frequently for single days) from sunshine figures supplemented by knowledge of latitude, season, and the solar constant. Measurement of reflexion is not quite so easy. In calculations Schmidt's value of 5% for open water and Ångström's value of 20% for green vegetation are used. Recent measurements by Pasquill[2] have confirmed that the latter is reasonable. Measurement of back radiation is difficult. Empirical formulae, based on such few measurements as exist, have been given by Ångström[3] and by Brunt.[4] Both show the net outflow as increasing with air temperature, with dryness of the air, and with clearness of the sky. In calculations Brunt's equation (the easier to handle) is preferred, using duration of bright sunshine to estimate clearness of the sky. Heat transfer to the air is estimated indirectly using a transport constant dependent upon wind speed in a way determined by experiment. Pasquill[2] has suggested that this constant is too small, but it could be 50% in error without having a serious effect on the resultant evaporation estimate. In the end it becomes possible to estimate the potential evaporation from a fresh green area of vegetation with an adequate water supply from knowledge of duration of bright sunshine, mean air temperature, mean vapour pressure and mean wind speed, supplemented by seasonal factors and constants obtainable from standard sources.

As a contrast to this diagram of the balance for an average summer day in a particular year, Fig. 2 shows the annual cycle for the Stour catchment area based on 10-yr monthly means.[5] This is given without any detailed explanation other than the following: (*a*) All energy is in evaporation units (1 mm = 59 cal/cm^2) and the symbols are as in Table 1. (*b*) The income $R_I = R_c(1 - r)$, i.e. is the residue after allowance for 20% reflexion except in July, August and September when the factor was increased to 25% to allow for extra reflexion from ripening crops. (*c*) The balance was drawn up in a way that left heat transfer to the air as the only unknown, and it is encouraging to find that its annual cycle is in phase with Best's[6] measurements of temperature gradients (ΔT_a) over similar crop and soil. It will be noted that it is only in summer that evaporation is the major sink for solar energy, and that estimation of evaporation in winter cannot be as precise as in summer, but fortunately the same fractional efficiency is not needed in winter because the absolute amount is so small.

About 90% of the year's evaporation takes place in the six months from April to September, amounting in southern England to about 18 in: for the winter six months the total is about 2 in.

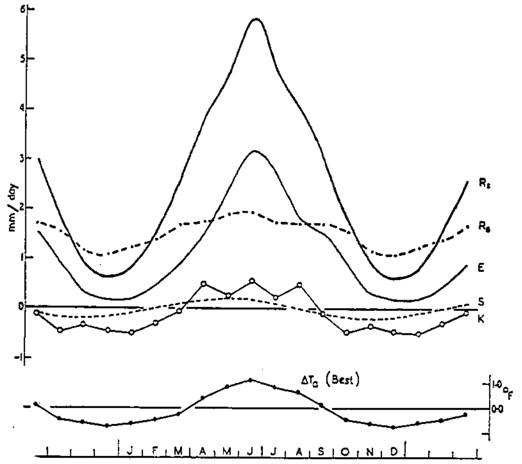

Fig. 2. Energy balance for grassland—based on ten-year monthly means. $R_I - R_B = E + S + K$ (1 mm/day = 59 cal/cm^2/day)

SOIL MOISTURE DEFICIT AND RIVER FLOW

Out of this demonstration of the important part played by vegetation in the meteorology of the air-soil boundary two ideas emerge to be carried forward. First, vegetation transfers large quantities of water from soil to air: and, second, these quantities can be estimated from weather data. The former has important consequences for soil and plant; the latter offers the possibility of interpretation, prediction and control.

The two important sets of soil conditions of practical interest are usually found side by side in most river valleys in which the stream is merely an exposed portion of the underground water-table. Close to the stream the water-table (well level) is near the surface of the ground, and in such an area plants draw their water from a layer of soil in hydrostatic contact with the water-table. This layer, in turn, is re-charged by upward movement from the water-table, so that maintained steady drying conditions at the surface usually produce a steady fall in the level of the water-table. Away from the stream the water-table is so far below the surface that plant roots cannot reach the layer in hydrostatic contact with it, and in such places all water withdrawn by plants must come from the soil without any re-charge from below, i.e. the soil in the root zone gets progressively drier without having any effect on the water-table. It is convenient to refer to the amount of this drying as the "soil moisture deficit" or, more simply, the "deficit," the level of zero deficit being that at which the soil can hold no more water against gravity. This is an arbitrary level, but has the practical advantages that it is a state fairly easily recognized in the field because it is the state in which through drainage first starts and at which it comes to an end, and it is a state that can be reproduced under controlled laboratory conditions. The study of natural evaporation is largely a study of short and long period changes in deficit: in agriculture it is important because when it becomes too big plant growth stops; in hydrology it is important because while it exists there is no appreciable movement of rainwater down to the water-table to replenish underground stores; and in soil mechanics it is important because of the changed physical properties of the soil, including actual shrinkage of some clay soils.

The dry weather flow of the stream is maintained by lateral drainage from the catchment. Away from the stream this causes a steady fall in the water-table, a fall not affected by surface wettings and dryings as long as there is a deficit in the soil above, but near the stream this underground current passes through a region in which deficits are negligible. Here, in effect, transpiring plants short circuit the current and the more active the transpiration the less the water reaching the stream, i.e. stream levels ought to fluctuate in anti-phase with transpiration. Observed day-to-day fluctuations in dry weather flow of streams are almost certainly a result of corresponding changes in transpiration rates, in turn resulting from day-to-day changes in weather, and a clear demonstration of the effect has been obtained on one of the streams of the Coweeta catchment in Asheville,[7] North Carolina, where continuous recording of stream level showed a marked diurnal oscillation, the level being higher at sunrise than at sunset. Although of no great practical importance, the phenomenon is a useful confirmation of the ideas from which it can be predicted, for it is known that transpiration rates go through a daily cycle in phase with meteorological factors that control it. In this same area near a stream there is a converse effect of a general state of near-zero deficit. There is little lag in response to rain, so that summer rainstorms produce a rapid local rise in the water-table and hence in stream level, and these short-lived summer spates come and go without in any way affecting the steady fall in well level at places away from the river bank.

CONTROL OF THE DEFICIT BY IRRIGATION

The important effects of plant transpiration are found where deficits can be built up. As the deficit increases the soil reaches a state of dryness at which extraction of water by the plant ceases to be easy, the transition being sharp in some soils and gradual in others, i.e. there is a value of the deficit beyond which the transpiration rate no longer is as great as the potential rate calculable from weather data. At this stage the growth rate will be reduced and very probably even before, so that in the interests of food production it is desirable to maintain the water supply at a level sufficient to maintain full transpiration, and an even greater supply may be needed to maintain full growth. Although guesses can be made at the value of the permissible upper limit, the only safe answer is to be obtained experimentally, crop by crop, and a start has been made in experiments run by tl British Sugar Corporation on the irrigation of sugar bee experiments carried out on commercial farms where tl farmers are already using irrigation systems. Ignorin fertilizer aspects of the experiments, there have bee either three or four groups of plots: (1) a control plo left to natural rainfall; (2) a "farmer's" plot which h irrigates when his experience suggests that it is needed (3) and (4) one (or two) plots which are irrigated on th basis of weather data collected on the field. The equip ment set up includes screened temperature and humidit recorders, an anemometer and a raingauge. Sunshin records are obtained from the nearest official observatory

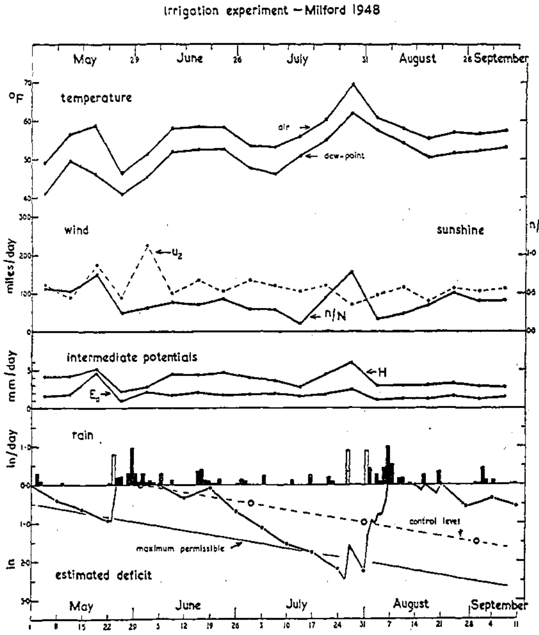

Fig. 3. Physical data for irrigation experiment, 1948
[*Reproduced from Quarterly Journal Royal Meterological Society*

Data are collected weekly and sent to Rothamsted Experimental Station, and the balance of evaporation and rainfall for the week estimated. Fig. 3[8] gives the physical history of the 1948 experiment, showing the week-by-week values of the four main weather elements and the two intermediate potentials calculated in estimating the week's evaporation. Weekly rainfall (closed column) and irrigation (open column) are also shown. The dotted line joining open circles shows the level of deficit aimed at throughout the season: below it, a full line represents the lower limit of tolerable deficit, the spacing being determined by the amount of water that could conveniently be applied in one day's work. The trend of deficit is shown by the full line joining full points. This season was not perfect, for on two out of three occasions rainfall was heavy after irrigation, and at the end of August it was predicted that irrigation would have no major effect on crop yield. This was so, as Table 2 reveals, but the weather was more helpful for the two experiments carried out in 1949, a magnificent year for this kind of experiment, and only marred by a severe virus disease infection that cut down all yields enormously. Briefly the story of the table is as follows. Treatment *O* is the control—natural rain only. Treatment *F* and treatment *J* are farmer's plots. (Note the great difference between the two in 1949. In terms of irrigation water, *J* had 4 in irrigation and *F* had 13 in irrigation.) Treatments *R* and *M* are based on weather data, and a guess at the order of magnitude of maximum permissible deficit: as it has happened, the guess has been good, for in all three experiments the weather-controlled treatments have given maximum yield. At Kesgrave the application of 6 in of water (treatment *M*) pushed up the yield of sugar by 60%.

Table 2. *Irrigation of sugar beet*

Yield of sugar (cwt/acre)—mean of all fertilizer treatments

Year	*Site*	*Treatment*	*Deficit on 31 August (in)*	*Yield*
1948	Milford	*F*	− 1·6*	64
		R	0·5	66½
		O	1·4	65½
1949	Milford	*F*	− 3·0*	41½
		R	1·5	44½
		M	3·0	43
		O	(7·0)	34½
1949	Kesgrave	*M*	2·8	49½
		J	4·3	46
		O	(6·5)	31

* Treatment *F* had too much water in both years. The negative deficit represents the estimated loss as drainage.

DEFICITS PRODUCED WHEN WATER SUPPLY IS LIMITED

The calculation of soil moisture deficit from weather data is only valid when there is adequate water available to meet the potential demand, a condition deliberately produced in the irrigation experiments except for treatments *O*. For these the estimates of Table 2 are much less certain: hence the brackets around them. The normal condition in large areas of the world, including south-east England, is that summer water supply is not sufficient to maintain maximum transpiration and growth. The plant's first main response to water shortage is to extend its root system, so tapping a deeper layer of soil, but there are limits to the rate and extent of this growth and a stage is reached at which the plant must be satisfied with what it can draw from the soil remote from the roots, and because water movement in a drying soil is so very, very slow, the transpiration rate falls off very abruptly to about one-tenth of what it could be. From laboratory data on the later stages of the drying of soil under constant drying conditions it has been possible to construct a synthetic curve connecting actual transpiration with potential transpiration,[9] so that seasonal changes in soil moisture deficit can be estimated from weather data even beyond the stage at which soil moisture becomes a limiting factor in transpiration rate. To construct the curve it is necessary to know how much readily available water can be held within the depth of rooting, and, fortunately, there is sufficient known about the rooting habit of most plants to be able to make an intelligent guess at this "root constant." The concept of a "root constant" is speculative, and has been introduced as a guide to future research rather than as a statement of what is true, but it is encouraging to find that no choice of the best value of the constant made on physical grounds has proved to be biologically unreasonable. Given the weather data, including rainfall, and knowing the dominant plant type, it should now be possible to make a quantitative study of the water balance of any cropped area. One example was given in the earlier survey, where it was shown that by calculating from the last date in spring when there was zero deficit (drains just stopped running) it was possible to predict the first autumn or winter running of field drains to within a few days. As the technique is successful for a single field, it should work for a whole catchment.

WATER BALANCE OF STOUR CATCHMENT AREA

Based on data supplied by a number of authorities a test has been made on the Stour (Suffolk and Essex) catchment month by month from 1933 to 1948.[10] The catchment lies mainly on the chalk of the north-east Chilterns, its area is about 330 square miles and it is roughly elliptical, the extensions of the long axis passing through Cambridge and Felixstowe. Monthly rainfall and river flow figures were available, and monthly values of potential transpiration were calculated from weather data recorded at Cambridge and Felixstowe. Estimates of actual transpiration were made by assuming the area to be in three parts: (*a*) an area near water-courses in which plants took their water direct from the water-table; (*b*) an area of deep-rooted vegetation for which a root constant of 8·0 in was used; and (*c*) an area of shallow-rooted vegetation for which a constant of 5·0 in was used (based on other experience of chalk soils). The water balance for any period is: rainfall is equal to the sum of evaporation, run-off, and increase in storage, the last representing the change in the quantity of water in the chalk below well level. As this was not measured it was made the unknown in the equation, rainfall and run-off being measured directly and evaporation estimated from weather data. Although this meant that an ideal direct check on the calculations was impossible, an adequate indirect check was obtained from well-level records at a site on the top of the same chalk ridge about 40 miles to the west of the catchment area. It was a very deep well, so a phase lag was to be anticipated, and it was sufficiently far away for its rainfall regime to be significantly different in some years, but in spite of these defects it was judged good enough to use. Fig. 4 shows the plot of observed well level, and of estimated storage calculated from an arbitrary datum of 10 in on 31 March, 1933. Although the two curves are out of phase—as expected—it is obvious that the main features of the annual cycle and the long term trends have been adequately reproduced in the estimated storage curve. Study of the diagram will reveal a discontinuity between the upper and lower halves, the relative positions of the

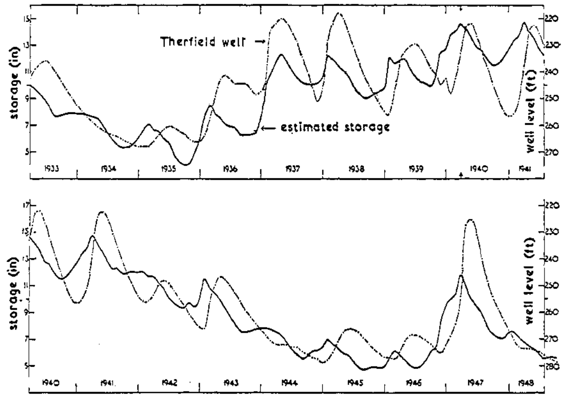

Fig. 4. Estimated changes in storage and observed changes in well-level: Stour 1933–48
[*Reproduced from Journal Institution of Water Engineers*

storage and well-level scales having been displaced by 10 ft on the well-level scale. The reason for the shift was a desire to free assessment of the goodness of fit in the second period from the effects of possible errors in the first, errors which are cumulative and include known differences in rainfall at the well and in the catchment during the first period. Some months after the completion of Fig. 4 a report came from the well site at Therfield stating that the measuring line had shrunk by 13 ft since it was last checked in 1937: the method of plotting virtually assumes that 10 ft had gone by 1941, but because it is not known how the 13 ft are to be spread over the period 1937–48, the diagram has been left untouched.

Two kinds of mean can be taken from the data. If the 15-yr monthly means are plotted, so giving a picture of the annual cycles, the two curves have much the same shape but the well curve lags about two months out of phase with estimated storage. If the lag is interpreted as the time taken for water to percolate 250 ft, then the average rate is about 4 ft/day, a very reasonable value for chalk. If the average annual values are plotted, based on a storage year from April to March, and a well year from June to May, so giving a smoothed picture of the long term trends already apparent in Fig. 4, a striking discontinuity is revealed round about 1938, from which it can be inferred that most, if not all, of the change in the line had taken place within a year of the 1937 check. Further analysis on an annual basis reveals an important relationship on which the next section is based: the 15-yr mean value of potential evaporation, as calculated, was 21·5 in/annum, whereas the actual annual mean, : estimated and as actually measured by difference of rai fall and run-off, was 20·0 in. The change affected t use of the synthetic curve is obviously not very grea and as these values are for the driest part of the Britis Isles, it seems reasonable to suppose that elsewhere similar estimate of potential transpiration (withoı correction for large deficits) should come to within 5 ‹ 10% of the actual evaporation. So, having stepped u from field to catchment area, the next stage is to consid the whole of the British Isles.

EVAPORATION OVER THE BRITISH ISLES

From four basic weather elements, regularly recorde in the Monthly Weather Report, it is possible to calculat potential transpiration: from measured values of rair fall and run-off it is possible to get a direct estimate c the actual transpiration. Weather data for 100 station were collected and average annual evaporation wa estimated for each from the long term averages: thes values were mapped without correction, knowing tha in south-east England they would be over-estimates o

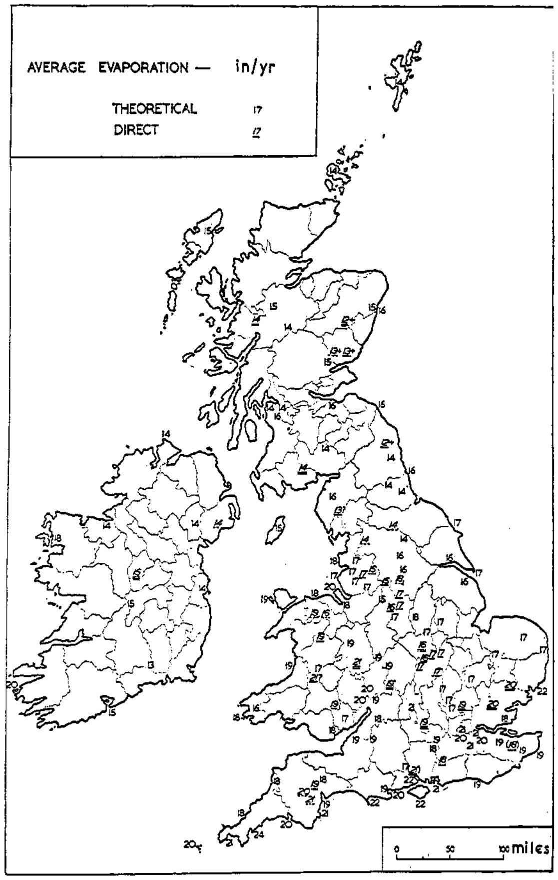

Fig. 5. Average annual evaporation over the British Isles
[*Reproduced from Quarterly Journal Royal Meterological Society*

actual evaporation. From official and private sources mean annual values of rainfall and run-off were obtained for 40 catchment areas, ranging in size from 4 to 4 000 square miles: in the absence of deep percolation and changes in storage the mean difference for each catchment should be the mean annual evaporation. Plotted on the same map (Fig. 5) the two sets of data agree extremely well.[11] Theoretical values are in upright figures, not underlined: direct estimates are in sloping figures, underlined to show order of size of area, a dashed line for less than 10 square miles, one full line for 10 to 100, two lines for 100 to 1 000 and three lines for over 1 000 square miles. There are too many minor sources of error to permit discussion of accuracy and it must suffice to state that agreement within 10% would have been acceptable: it is pleasing to find that the general agreement is within 5%. As the two sets of estimates have nothing in common (air temperature, vapour pressure, wind and sunshine in one: rainfall and run-off in the other) their agreement is strong presumptive evidence that both are true and that this physical basis of estimating evaporation can be applied to parts of these islands where direct values do not exist.

The map can be regarded as a contribution to the climatology of the British Isles, or as an indication of the average annual water requirement for agriculture, or as showing the average annual loss that water engineers must allow for in planning impounding operations. None of these will be discussed here, and the only application to be made is one that could have followed the description of the irrigation experiments.

EFFECT OF VEGETATION ON CLAY SOILS

It will be noted that over south and south-east England the annual evaporation in an average year is 17 to 20 in, and as about 90% takes place from April to September, the potential summer evaporation is about 15 to 18 in, i.e. in dry summers when the seasonal rainfall may be only 6 to 9 in, the potential deficit that could be built up in the six months is about 9 in. This is the deficit that would be reached under trees, but under grass with its shorter root system the limit would be about 6 in. The withdrawal of this water affects many properties of the soil, irrespective of soil type, but for clay soils there is a further particular effect of considerable practical importance, and as such soils cover much of south-east England the effect is widespread. When these clay soils are saturated (i.e. at zero deficit) the porespace is full of water: as they dry the micro-porespace remains full of water, i.e. as water is removed it is not replaced by air, but, instead, the clay particles come closer together. This shrinkage of the aggregates necessarily involves the production of cracks between the aggregates and evidence of this can be found on almost any lawn in a dry summer. If the shrinkage is isotropic (it probably is nearly so) there will be an equal movement vertically, and for a deficit of 6 in. in a dominantly clay soil the vertical movement of the surface should be about 2 in. At the Building Research Station gauges have been designed to measure the vertical movement, and in Fig. 6 is a plot of the movements at several depths under grass-covered soil during the dry summer of 1947.[12] Extrapolation to find the surface movement gives a maximum value of about 2 in, which is what the meteorological and soil physics of the process predicts that it ought to be. A

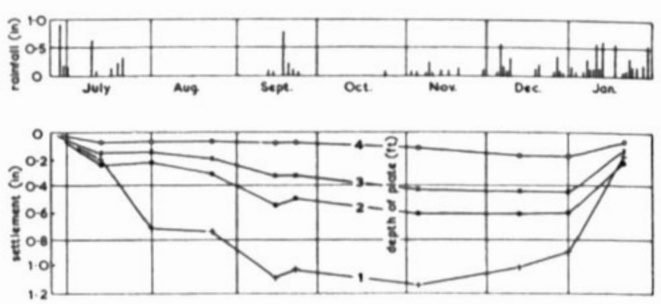

Fig. 6. Vertical movements of clay soil at Garston, 1947 [*Crown Copyright reserved*

Fig. 7. Damage to a house, caused by shrinkage of clay soil, Edgware, 1947 [*Crown Copyright reserved*

house or a road on such a soil will not suffer if the shrinkage is the same everywhere under the foundation, but the mere act of covering the ground eliminates transpiration inside the area without stopping it round the edges, where there may be a lawn or flower bed. But many houses and roads have the aggravating circumstance of neighbouring trees, particularly poplar trees, which throw out a long shallow horizontal root system that may be 100 ft in length. So under the middle of the house or the crown of the road there is little seasonal change in the moisture content and little change in volume: at the edges there is a seasonal cycle of alternate shrinkage and swelling, and eventually, of course, under this differential stress something gives way. Fig. 7 shows the effect on a house, and during 1947 and 1949 similar structural damage occurred in many buildings whose foundations were not deep enough to be below the main drying action of tree roots. The effect on roads may take the form of cracking (especially round manholes) or of tilting of slabs of road material.(13) On clay soils it is obvious that the amenity value of trees in gardens and by roadsides may often be obtained only at a rather high cost, particularly if the trees are poplars.

CONCLUSION

Although the survey has dealt almost exclusively with the part played by plants in the transfer of water across the earth-air boundary, it should be sufficient to demonstrate the great importance of vegetation as a factor in open air physics, pure and applied. Much remains to be done and the problems are sufficiently complex to present a worthwhile challenge in fundamental physics, the successful acceptance of which would be of great benefit to agriculture, meteorology, hydrology, soil mechanics and several other branches of applied science.

REFERENCES

(1) PENMAN, H. L. *J. Sci. Instrum.*, **25**, p. 425 (1948).
(2) PASQUILL, F. *Proc. Roy. Soc. A*, **198**, p. 116 (1949).
(3) ÅNGSTRÖM, A. *Geog. Ann.*, **2**, p. 253 (1920).
(4) BRUNT, D. *Physical and Dynamical Meteorology* (London: Cambridge University Press, 1934).
(5) PENMAN, H. L. Paper read at Brit. Assoc., Newcastle (1949).
(6) BEST, A. C. *Geophysical Memoir* No. 65 (London: His Majesty's Stationery Office, 1935).
(7) WISLER, C. O., and BRATER, E. F. *Hydrology* (New York: John Wiley and Sons Ltd., 1949).
(8) PENMAN, H. L. *Quart. J. Roy. Met. Soc.*, **75**, p. 293 (1949).
(9) PENMAN, H. L. *Brit. J. Soil Sci.*, **1**, p. 74 (1949).
(10) PENMAN, H. L. *J. Instn. Water Eng.*, **4**, p. 457 (1950).
(11) PENMAN, H. L. *Quart. J. Roy. Met. Soc.*, **76**, p. 372 (1950).
(12) WARD, W. H. *Biology and Civil Engineering*, p. 181 (London: Institution of Civil Engineers, 1949).
(13) CRONEY, D., and LEWIS, W. A. *Biology and Civil Engineering*, p. 195 (London: Institution of Civil Engineers, 1949).

ORIGINAL CONTRIBUTIONS

A theory of stresses in glass butt seals

By H. RAWSON, B.Sc.Tech., Research Laboratories, The British Thomson-Houston Co. Ltd., Rugby

[*Paper received 5 December*, 1950]

A series of equations are derived which may be used for the calculation of the stresses in glass butt seals and three component graded seals between materials of differing expansion coefficients. Stress determinations were made by the photoelastic method and good agreement was found between the theoretical and experimental values.

The theory set out in this paper deals with the magnitude and the distribution of stresses in glass seals in which two tubes of materials of different expansion coefficients are sealed end to end at a high temperature and then cooled down to a lower temperature at which both materials are rigid. In practice both tubes may be glass or one may be metal. The theory is later developed to deal with seals consisting of more than two components. The graded seal shown diagrammatically in Fig. 1 is an example of this. In a graded seal the end tubes of widely different coefficients are joined by a series of short lengths of glass of intermediate expansions.

THE STRESSES IN SIMPLE BUTT SEALS

Fig. 2 shows a butt seal in which two cylinders, 1 and 2, are sealed end to end at A. The tubes are made from materials of different expansion coefficients and it is assumed that they are of semi-infinite length; the cylinders have the same mean radius R and wall thickness t. The sealing is carried out at a high temperature after which the seal is cooled; it is assumed that there is a temperature T_1 above which the seal is stress free and below which any difference in the rate of contraction of the two materials causes stresses to be set up in the tubes.

The seal is cooled to a temperature T_2 below T_1, and

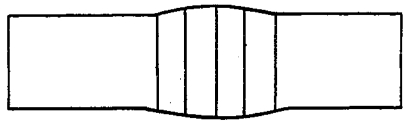

Fig. 1. Graded seal

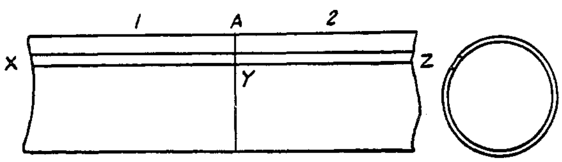

Fig. 2. Butt seal

it is required to find the stresses in the seal at this temperature, T_2. It is assumed for simplicity that the expansion coefficients α_1 and α_2 of the two materials, 1 and 2, are constant over the temperature range T_1 to T_2.