

The result of this theorem is used in John and Smith (1972, equation (4.6)) and in Rao and Mitra (1971). The restrictive nature of Pringle and Rayner's result appears to be due to their failure to use the consistency condition for the hypothesis, namely that \mathbf{z} should be in the column space of \mathbf{L} . Since the class of g_1 -inverses is far wider than the class of g_2 -inverses a wider choice of methods for testing non-full rank linear hypotheses is available. For instance in a number of practical situations the matrix \mathbf{LGL}' is idempotent and a g_1 -inverse is given by the identity matrix which considerably simplifies subsequent analysis. We note that the identity matrix is not in general a g_2 -inverse of an idempotent matrix.

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Alternative Derivation of the Sum of Squares in a Non-full Rank General Linear Hypothesis

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[Received June 1973]

Keywords: SUM OF SQUARES; LINEAR HYPOTHESIS; NON-FULL RANK; GENERALIZED INVERSE

IN the preceding note, John and Smith (1974) show that the g_2 -inverse in their equation (1) can be replaced by a g_1 -inverse, thus resolving an apparent inconsistency between Pringle and Rayner (1971) and John and Smith (1972). Their proof used various properties of generalized inverses, but the following proof may be more intuitively appealing to some readers.

I use $M(\mathbf{A})$ to denote the column space of any matrix \mathbf{A} .

From elementary considerations it is clear that for a function of $\boldsymbol{\beta}$ to be estimable, it must be a function of $\mathbf{X}\boldsymbol{\beta}$, the expectation of \mathbf{y} . Hence the condition that $\mathbf{L}\boldsymbol{\beta}$ is estimable implies that $M(\mathbf{L}) \subseteq M(\mathbf{X})$, which in turn implies that $\mathbf{L} = \mathbf{A}\mathbf{X}$ for some matrix \mathbf{A} . We may suppose without loss of generality that $M(\mathbf{A}') \subseteq M(\mathbf{X})$ (for, if necessary, we may subtract from each row of \mathbf{A} the component orthogonal to $M(\mathbf{X})$). Then, since $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-}\mathbf{X}'$ is the orthogonal projection onto $M(\mathbf{X})$ (where $(\mathbf{X}'\mathbf{X})^{-}$ is a g_1 -inverse of $\mathbf{X}'\mathbf{X}$), we have

$$\begin{aligned} \mathbf{LGL}' &= \mathbf{A}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-}\mathbf{X}'\mathbf{A}' \\ &= \mathbf{A}\mathbf{A}'. \end{aligned}$$

Then

$$\mathbf{L}'(\mathbf{LGL}')^{-}\mathbf{L} = \mathbf{X}'\mathbf{A}'(\mathbf{A}\mathbf{A}')^{-}\mathbf{A}\mathbf{X}$$

which does not depend on the choice of $(\mathbf{LGL}')^{-}$ since in any case $\mathbf{A}'(\mathbf{A}\mathbf{A}')^{-}\mathbf{A}$ is simply the orthogonal projection onto $M(\mathbf{A}')$.

Following John and Smith (1974) and writing $LL^{-1}z$ for z in their equation (1) we then see that $(LGL')^{g_2}$ can be replaced by an arbitrary g_1 -inverse.

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 Corrigendum

A Bayesian Significance Test for Multinomial Distributions

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J. R. Statist. Soc. B, **29**, 399–431

Formula (32) is incorrect: the integral diverges if $\phi(k) > 1/k^{(t+1)/2}$ for large k . The error arose because (29) is true for fixed k when $N \rightarrow \infty$, but not for fixed large N when $k \rightarrow \infty$. The remark following equation (33) is true only when N is large enough.

On page 411, for “ G has a density” read “ G^2 has a density”.

Neither of these errors affects the rest of the paper.