The analysis of field experiments on an electronic computer

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From the point of view of the numerical analyst, the statistical analysis of the results of a field experiment is a comparatively trivial task. However, the relative volume of input and output bring the problem more into the category of data-handling, and several problems arise in the preparation of the figures for the machine and in the organisation of the printed results in such a way as to make the presentation and interpretation as simple as possible for the experimenter himself.

The Elliott/N.R.D.C. 401 computer has been in use at Rothamsted for the past five years, and during 1958 was used, among other work, for the analysis of 1664 field experiments with an average of 3, 4 variates per experiment. It will be clear that the efficient handling of this volume of figures has necessitated careful attention to the organisation of the work at all stages. This article summarises some of the principles on which our programmes are based. A fuller description will be found in Yates, Healy and Lipton (1957) and a general discussion of the use of electronic computers in research statistics in Yates and Rees (1958).

The computer itself is of medium size, with a storage capacity of 2944 words, each of 32 binary digits (approximately $9^1/_2$ decimal digits). Input and output are normally on 5-hole punched paper tape, providing a range of 60 numerical and alphabetical characters.

Input of Data

It is not always recognized that the complete process of analysis often involves as much preliminary computation on the original figures as statistical analysis in the ordinary sense. As an example, we may consider an experiment on sugarbeet, where it is usual to record in the field, for each plot,

A Weight of dirty roots (lb/plot)
B Weight of dirty sample (lb.)
C Weight of clean sample (lb.)
D Weight of tops (lb/plot)

and in the laboratory:

E Sugar content, %

and possibly other determinations. The quantities required for analysis are:

F Weight of clean roots (cwt/acre)
G Sugar content, %
H Weight of sugar (cwt/acre)
I Weight of tops (cwt/acre)

If λ is the conversion factor from lb/plot to cwt/acre, we have:

$$F = \lambda \cdot A \cdot C/B$$
$$G = E$$
$$H = E \cdot F$$
$$I = \lambda \cdot D$$

These computations must be carried out for each plot in the experiment. They are very simple, but when undertaken by hand they are time-consuming and also liable to error. Accordingly, it is convenient to include them in the complete analysis programme, and for this purpose we have constructed a general programme which can be incorporated with only minor changes in the programme for any particular design. In addition to carrying out these preliminary computations, this general programme takes care of the actual input and storage of the data; it also records a heading from the input tape and copies it on to the output tape so that no difficulties of identification can arise. The user writes the instructions for this programme in algebraic notation. Thus in the example above, after the original data had been placed in sections of the store numbered 0–4, it would be necessary to write:

$$\lambda = 0.2468$$
$$1 = 2/1$$
$$0 = 0 \cdot 1$$
$$0 = 0 \cdot \lambda$$
$$1 = 4$$
$$2 = 0 \cdot 1$$
$$3 = 3 \cdot \lambda$$

in order to form variates F–I in sections 0–3 of the store. Provision is made for the scaling of products and quotients by suitable powers of 10 when required. The programme forms at each stage the maximum, minimum and mean of the set of results and these are printed out as a visual check. The mean and the total can also be stored for future reference.

Analysis

The programme for the actual statistical analysis will clearly depend upon the design of the particular experiment. Considerable generality can be

achieved (see Tocher, 1952; Hartley, 1956; Tocher, 1957), but a very general programme will inevitably be long and usually not of full efficiency in any particular case. At the time of writing, the programmes at Rothamsted consist of:

1. Randomized blocks (up to 126 plots)
2. Latin squares (up to 10×10)
3. $p \times q$ factorial experiment in randomized blocks
4. $p \times q$ factorial experiment in split plots
5. 2^n factorial experiment (up to 128 plots)
6. 3^3 factorial experiment (single replicate)
7. General factorial experiment (up to 6 factors, each with up to 8 levels).

Programme 5 will handle total and partial confounding, fractional replication and 8×8 quasi-Latin squares. Programme 6 is the least general, but the 3^3 single replicate experiment is widely used in fertiliser research and represents a large proportion of the total number of experiments analysed. Programmes 1–4 could be readily combined into a single programme, and it is hoped to do this shortly. A programme for incomplete block designs has so far not been justified by the demand for this type of analysis.

Hartley (1956) has suggested the useful device of working with deviations from expected values. This is used in programme 5 for the analysis of 2^n factorial experiments, the first stage being the formation of block means and the replacement of the original yields by deviations from block means. This has the effect of setting all confounded contrasts equal to zero. At a later stage, in this and all other programmes, the ultimate residuals from block and treatment means are formed and stored. These have many uses, some of which are mentioned later in this article. The original yields are stored in randomized order as received from the field, and this makes it possible to print out the residuals in "geographical" order and to examine them visually for irregularities when the accuracy of an experiment is lower than usual. The formation of the residuals also makes possible the direct computation of the residual sum of squares, and the additive property of the analysis of variance then provides a valuable overall check. Other checks can be carried out by forming block and treatment totals of the residuals, all of which should vanish, apart from rounding errors.

The action to be taken when a check fails requires some consideration. At first sight it appears that, after a check failure, further "results" will be useless and that the computations should be stopped at once. However, some checks are only approximate, and liable to occasional failure through an unfortunate combination of rounding errors. When such checks fail, it

may be preferable to print out the magnitude of the discrepancy and to proceed with the computation.

On the failure of any check, it is important that some indication be made in the printed record. This will prevent incorrect results from being sent out, but the indication should also provide some information on the nature and extent of the failure that has occurred. It is usually undesirable for the machine operator to have to investigate the cause of a failure. She will normally proceed to the next experiment, or, if repeated failures occur, call in the technical staff.

The formation of sums of squares is liable to involve large numbers which may grow outside the capacity of a single storage location. Various methods are available to overcome this. A working mean can be used, as in desk computation; the computer has no special preference for simple numbers, so the working mean can be the actual mean, calculated with a suitable number of guard figures. Alternatives are the calculation of:

$$n \Sigma (x - \bar{x})^2 = n \Sigma x^2 - (\Sigma x)^2$$

or

$$n^2 \Sigma (x - \bar{x})^2 = \Sigma (n x - \Sigma x)^2$$

both of which give exact results. Users should also be dissuaded from presenting their results with an unjustifiably large number of significant figures. Electronic computation does not of itself increase the accuracy of the observations!

Output

Output of the results of an analysis usually takes considerably longer than the analysis itself and may also involve more machine instructions. It is of great importance, nevertheless, that the printed record should be organised into a layout that is easily interpretable by the experimenter without having to be labelled by hand before being sent out. I have mentioned above the provision of a heading, which may include a description of the variates analysed. The machine can also be made to print a $\pm$ symbol before a standard error and to label the items in an analysis of variance. The provision of such labels is usually expensive in terms of storage space, and the amount provided will depend on the facilities available.

The amount of material printed should be to some extent under the control of the user. Thus the Rothamsted programmes can print out:

a) the actual yields
b) the residuals
c) the analysis of variance table
d) the treatment means and appropriate standard errors.

Item (d) will presumably be required for all variates analysed, but any of the other items can be suppressed by suitable indications on the data tape, leading to an appreciable saving in time.

Special features

1. *Split plots*

A special programme is used at Rothamsted for the analysis of randomized blocks with split plots, but other main-plot designs can be handled with the aid of the general input programme. Each level of the sub-plot factor is taken in as a separate variate. The general input programme is used to replace these variates by an orthogonal set of contrasts, the contrasts are analysed separately and the analyses pooled in the appropriate manner. When the main plots are split into two, the analysis of sums and differences in this manner provides a virtually complete solution.

2. *Covariance*

The complete analysis of covariance presents some awkward problems for automatic computation, since the size of the regression coefficient cannot easily be predicted in advance. It is, however, simple for the machine to carry out the most arduous part of the work, namely the formation of the residual sum of products. With our programmes, this is readily formed as the sum of products of the residuals of one variate with the yields or resi duals of the other.

3. *Missing values*

Missing values are liable to occur with any design and it is important to have a technique for handling the analysis which is independent of the details of the design. The technique used at Rothamsted (Healy and Westmacott, 1956) consists in starting with guessed values for the missing plots and in forming the residuals by the normal programme; the residuals on the missing plots are then subtracted from the guessed values and the process iterated until the residual sum of squares fails to decrease. The advantage of this process is that most of the work is done by the regular programme. It has been found to be adequately fast, and is particularly efficient when several plots are missing.

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